MATH 1101Y 2009 Quiz 11 (a)

1. (2 pts) Find the area of the region enclosed by the curves $y = x^2 - x$ and $y = 3x$.

   Solution: First we find the intersections of these two curves. We let

$$\begin{aligned} x^2 - x &= 3x \\ x^2 - 4x &= 0 \\ x(x-4) &= 0 \end{aligned}$$

$$x = 0 \text{ or } x = 4$$

   Let $x = 2$. $x^2 - x = 2$ and $3x = 6$. The curve $y = 3x$ is on top. We have

$$\begin{aligned} A &= \int_0^4 \left(3x - \left(x^2 - x\right)\right) dx \\ &= \int_0^4 \left(4x - x^2\right) dx \\ &= \left[2x^2 - \frac{x^3}{3}\right]_0^4 = 32 - \frac{64}{3} \\ &= \frac{32}{3}. \end{aligned}$$

□

2. (3 pts) Use the method of cylindrical shells to find the volume generated by rotating the regoin bounded by the curves $y = 2 - (x-2)^2$ and $y = 1$ about the y-axis.

   Solution: To find the intersections of these two curves, we let

$$\begin{aligned} 2 - (x-2)^2 &= 1 \\ (x-2)^2 &= 1 \end{aligned}$$

$$x = 1 \text{ or } x = 3$$

   Let $x = 2$. $2 - (x-2)^2 = 2$. The curse $y = 2 - (x-2)^2$ is above $y = 1$ for $x \in (1, 3)$. Using the method of cylindrical shells we have

$$\begin{aligned} V &= \int_1^3 2\pi x \left(2 - (x-2)^2 - 1\right) dx \\ &= 2\pi \int_1^3 x \left(-x^2 + 4x - 3\right) dx \\ &= 2\pi \int_1^3 \left(-x^3 + 4x^2 - 3x\right) dx \\ &= 2\pi \left[-\frac{x^4}{4} + \frac{4}{3}x^3 - \frac{3}{2}x^2\right]_1^3 \\ &= \frac{16}{3}\pi. \end{aligned}$$

□