Mathematics 110 – Calculus of one variable
§A FINAL EXAMINATION
Trent University, 23 April, 2002

Time: 3 hours *Brought to you by* Стефан Біланюк.

Instructions: Show all your work and justify all your answers. *If in doubt,* **ask!**

Aids: Calculator; an $8.5'' \times 11''$ aid sheet or the pamphlet *Formula for Success*; one brain.

Part I. Do all three of **1** – **3**.

1. Find $\frac{dy}{dx}$ (in terms of x and/or y) in any *three* of **a** – **f**. *[15 = 3 × 5 ea.]*

a. $y = \dfrac{x^2 - 1}{x^2 + 1}$ **b.** $y = \displaystyle\int_{-x}^{x} e^{4t}\,dt$ **c.** $\begin{array}{c} y = \cos(t) \\ x = \sin(t) \end{array}$

d. $y = \arcsin(3x)$ **e.** $x^2 + 2xy + y^2 = 1$ **f.** $y = \ln\left(x^2 - 2x + 1\right)$

2. Evaluate any *three* of the integrals **a** – **f**. *[15 = 3 × 5 ea.]*

a. $\displaystyle\int \frac{x+2}{x^2 + 4x + 5}\,dx$ **b.** $\displaystyle\int_0^\infty \frac{1}{t^2+1}\,dt$ **c.** $\displaystyle\int \frac{1}{x^2 - 5x + 6}\,dx$

d. $\displaystyle\int_1^e \ln(y)\,dy$ **e.** $\displaystyle\int \frac{1}{\sqrt{4 - x^2}}\,dx$ **f.** $\displaystyle\int_0^\pi \cos^2(w)\sin(w)\,dw$

3. Do any *five* of **a** – **j**. *[25 = 5 × 5 ea.]*

a. Does $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \arctan(n)}{n^2}$ converge absolutely, converge conditionally, or diverge?

b. Evaluate $\displaystyle\lim_{x \to 0} \frac{x}{e^x - 1}$ or show that the limit does not exist.

c. Find the arc-length of the curve given by $y = 1 - t^2$ and $x = 1 + t^2$ for $0 \le t \le 1$.

d. What is the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n}{n!}$ when it converges?

e. Sketch the region described by $0 \le r \le \csc(\theta)$ and $\pi/4 \le \theta \le \pi/2$ in polar coordinates and find its area.

f. Use an $\varepsilon - \delta$ argument to verify that $\displaystyle\lim_{t \to 3}(2x - 4) = 2$.

g. Determine the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n+1}}{2^n} x^n$.

h. For which values of c is $f(x) = \begin{cases} \cos(x) & x \le \pi \\ cx & x > \pi \end{cases}$ continuous at $x = \pi$?

i. Find the absolute maximum and minimum points, if any, of $f(x) = x^3 - 3x$ on the interval $-2 \le x \le 2$.

j. Give an integral corresponding to the Right-hand Rule sum $\displaystyle\sum_{i=1}^{n} \frac{2i}{n} \tan\left(1 + \frac{i}{n}\right)$.

Part II. Do *one* of **4** or **5**.

4. Find the domain, all maximum, minimum, and inflection points, and all vertical and horizontal asymptotes of $g(x) = e^{-x^2}$, and sketch its graph. *[15]*

5. Sand is poured onto a level floor at the rate of 60 l/min. It forms a conical pile whose height is equal to the radius of the base. How fast is the height of the pile increasing when the pile is 2 m high? *[15]*
[The volume of a cone of height h and base radius r is $\frac{1}{3}\pi r^2 h$.]

Part III. Do *one* of **6** or **7**.

6. Consider the curve $y = \sqrt{x}$, $0 \leq x \leq 4$.

- **a.** Sketch the curve. *[1]*
- **b.** Sketch the surface obtained by revolving the curve about the x-axis. *[2]*
- **c.** Find the area of the surface. *[12]*

7. Consider the region in the first quadrant bounded by $y = \sqrt{x}$ and $y = \dfrac{x}{2}$.

- **a.** Sketch the region. *[2]*
- **b.** Sketch the solid obtained by revolving the region about the y-axis. *[2]*
- **c.** Find the volume of the solid. *[11]*

Part IV. Do *one* of **8** or **9**.

8. Consider the power series $\displaystyle\sum_{n=1}^{\infty} (-2)^n n x^{n-1} = -2 + 8x - 24x^2 + 64x^3 - 160x^4 + \cdots$

- **a.** Find the radius and interval of convergence of this power series. *[9]*
- **b.** What function has this power series as its Taylor series at $a = 0$? *[6]*

9. Let $f(x) = e^{2x-2}$.

- **a.** Find the Taylor series at $a = 1$ of $f(x)$. *[10]*
- **b.** Find the radius and interval of convergence of this Taylor series. *[5]*

[Total = 100]

Part MMIII. Bonus!

-1. Write a little poem about calculus or mathematics in general. *[2]*

-2. Find the surface area of a cone with base radius r and height h. For maximum credit, do this without using any calculus. *[2]*

I HOPE YOU'VE HAD A GOOD TIME! HAVE A GOOD SUMMER!