

## Mathematics 110 – Calculus of one variable

Trent University 2001-2002

### SOLUTIONS TO THE BONUS ASSIGNMENT

#### THE UNEXPECTED TEST

In a certain mathematics class Professor B, who always tells the truth and is never mistaken, explains the marking scheme for the course to the students.

“This course meets once each week. There will be only one test, which will be written in class in one of the twelve weeks of the next term. However you will not know which week it is until the class in which the test is given.”

**Problem:** Is there any way to determine in which week the test is given? Why or why not? If so, in which week will the test be written?

**SOLUTION.** We’ll analyze a closely related paradox, mentioned in tutorial, that brings the underlying issues into sharper relief. In particular, it gets rid of the red herring in the original problem: the twelve weeks in which the test might be offered. (Just as well, since a twelve-week-old herring really stinks!)

#### THE UNEXPECTED TEST (CONDENSED VERSION)

In a certain mathematics class Professor B, who always tells the truth and is never mistaken, says to the students, “There will be a test in the next class. However, you cannot be sure until that class whether the test will be offered in it or not.”

Is there enough information here for the students to determine whether or not the test will be offered?

Well, no. In fact, from the students’ point of view the given information is actually contradictory because it boils down to:

- I will definitely have a test in the next class.
- Maybe not.

One can’t have it both ways<sup>†</sup> . . .

However, from the point of view of the professor or of an outside observer there is no contradiction. The reason that the professor’s second statement doesn’t conflict with the rest of the information in these frames of reference is because it applies to, and only to, the students: “ . . . *you* cannot be sure . . . ” That ‘you’ introduces a self-reference *for the students* leading to the contradiction noted above, but *not* for the professor or an outside observer. This makes it impossible for the students to determine whether the test will be offered or not, which, in turn, ensures the truth of the second statement in the professor’s or an outside observer’s frame of reference.<sup>‡</sup>

The solution to the original paradox is very similar and you should be able to work it out for yourself now. ■

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<sup>†</sup> Skipping off on the next class to avoid the situation doesn’t count!

<sup>‡</sup> Ironic, isn’t it? The fact that the second statement leads to a contradiction from one point of view makes it true from another . . .