Mathematics 110 – Calculus of one variable
Trent University 2001-2002

QUIZZES

Quiz #1. Friday, 21 September, 2001. *[15 minutes]*

1. Sketch the graph of a function $f(x)$ with domain $(-1,2)$ such that $\lim_{x\to 2} f(x) = 1$ but $\lim_{x\to -1} f(x)$ does not exist. *[4]*
2. Use the $\epsilon - \delta$ definition of limits to verify that $\lim_{x\to\pi} 3 = 3$. *[6]*

Quiz #2. Friday, 28 September, 2001. *[15 minutes]*

Evaluate the following limits, if they exist.

1. $\displaystyle\lim_{x\to -1} \frac{x+1}{x^2-1}$ *[5]* 2. $\displaystyle\lim_{x\to 1} \frac{x+1}{x^2-1}$ *[5]*

Quiz #3. Friday, 5 October, 2001. *[20 minutes]*

1. Is $g(x) = \begin{cases} \dfrac{x^2-6x+9}{x-3} & x \neq 3 \\ 0 & x = 3 \end{cases}$ continuous at $x = 3$? *[5]*
2. For which values of c does $\displaystyle\lim_{x\to\infty} \frac{13}{cx^2+41}$ exist? *[5]*

Quiz #3. *(Late version.)* Friday, 5 October, 2001. *[20 minutes]*

1. For which values of the constant c is the function $f(x) = \begin{cases} ce^x & x \geq 0 \\ c - x & x < 3 \end{cases}$ continuous at $x = 0$? *[5]*
2. Compute $\displaystyle\lim_{x\to\infty} \frac{(x+13)^2}{2x^2+\frac{1}{x}}$, if it exists. *[5]*

Quiz #4. Friday, 12 October, 2001. *[10 minutes]*

1. Use the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{5}{7x}$. *[10]*

Quiz #5. Friday, 19 October, 2001. *[17 minutes]*

Compute $\dfrac{dy}{dx}$ for each of the following:

1. $y = \dfrac{2x+1}{x^2}$ *[3]* 2. $y = \ln(\cos(x))$ *[3]* 3. $y = (x+1)^5 e^{-5x}$ *[4]*

Quiz #6. Friday, 2 November, 2001. *[20 minutes]*

Find $\dfrac{dy}{dx}$ ...

1. ... at the point that $y = 3$ and $x = 1$ if $y^2 + xy + x = 13$. *[4]*
2. ... in terms of x if $e^{xy} = x$. *[3]*
3. ... in terms of x if $y = x^{3x}$. *[3]*

Quiz #7. Friday, 9 November, 2001. *[13 minutes]*

1. Find all the maxima and minima of $f(x) = x^2 e^{-x}$ on $(-\infty, \infty)$ and determine which are absolute. *[10]*

Quiz #8. Friday, 23 November, 2001. *[15 minutes]*

1. A spherical balloon is being inflated at a rate of 1 m^3/s. How is the diameter of the balloon changing at the instant that the radius of the balloon is 2 m? *[10]*
   [The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.]

Quiz #9. Friday, 30 November, 2001. *[20 minutes]*

1. Use the Right-hand Rule to compute $\int_0^3 \left(2x^2 + 1\right) dx$. *[6]*

   [You may need to know that $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$.]

2. Set up and evaluate the Riemann sum for $\int_0^2 (3x+1) dx$ corresponding to the partition $x_0 = 0$, $x_1 = \frac{2}{3}$, $x_2 = \frac{4}{3}$, $x_3 = 2$, with $x_1^* = \frac{1}{3}$, $x_2^* = 1$, and $x_3^* = \frac{5}{3}$. *[4]*

Quiz #9. *(Late version.)* Friday, 30 November, 2001. *[20 minutes]*

1. Use the Right-hand Rule to compute $\int_1^2 (x+1) dx$. *[6]*

   [You may need to know that $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$.]

2. Set up and evaluate the Riemann sum for $\int_0^4 x^2 dx$ corresponding to the partition $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$, with $x_1^* = 0$, $x_2^* = 2$, $x_3^* = 2$, and $x_4^* = 4$. *[4]*

Quiz #10. Friday, 7 December, 2001. *[20 minutes]*

Given that $\int_1^4 x\, dx = 7.5$ and $\int_1^4 x^2\, dx = 21$, use the properties of definite integrals to:

1. Evaluate $\int_1^4 (x+1)^2\, dx$. *[5]*

2. Find upper and lower bounds for $\int_1^4 x^{3/2}\, dx$. *[5]*

Quiz #10. *(Late version.)* Friday, 7 December, 2001. *[20 minutes]*

1. Without evaluating them, put the following definite integrals in order, from smallest to largest. *[5]*

$$\int_0^2 \sqrt{x^2+1}\, dx \qquad \int_0^1 x\, dx \qquad \int_0^1 \sqrt{x^2+1}\, dx \qquad \int_0^1 x^3\, dx \qquad \int_0^2 (x+3)\, dx$$

2. Write down (you need not evaluate it) a definite integral(s) representing the area of the region bounded by the curves $y = x - x^3$ and $y = x^3 - x$. *[5]*

Quiz #11. Friday, 11 January, 2002. *[15 minutes]*

1. Compute the indefinite integral $\int \left(x^2+x+1\right)^3 (4x+2)\,dx$. *[5]*
2. Find the area under the graph of $f(x) = \sin(x)\cos(x)$ for $0 \le x \le \frac{\pi}{2}$. *[5]*

Quiz #12. Friday, 18 January, 2002. *[15 minutes]*

1. Compute $\int_1^e \frac{\ln(x^2)}{x}\,dx$. *[5]*
2. Find the area of the region between the curves $y = x^3 - x$ and $y = x - x^3$. *[5]*

Quiz #12. *(Late version.)* Friday, 18 January, 2002. *[15 minutes]*

1. Compute $\int_1^{\ln(2)} \frac{e^x}{e^{2x}+1}\,dx$. *[5]*
2. Find the area of the region bounded below by the curve $y = x^2 - 1$ and above by the curve $y = \cos\left(\frac{\pi}{2}x\right)$, where $-1 \le x \le 1$. *[5]*

Quiz #13. Friday, 25 January, 2002. *[19 minutes]*

1. Find the volume of the solid obtained by revolving the region in the first quadrant bounded by $y = \frac{1}{x}$, $y = x$, and $x = 2$ about the x-axis. *[10]*

Quiz #14. Friday, 1 February, 2002. *[17 minutes]*

1. Suppose the region bounded above by $y = 1$ and below by $y = x^2$ is revolved about the line $x = 2$. Sketch the resulting solid and find its volume. *[10]*

Quiz #15. Friday, 15 February, 2002. *[25 minutes]*

Evaluate each of the following integrals.

1. $\displaystyle\int_0^{\pi/4} \tan^2(x)\,dx$ *[4]*
2. $\displaystyle\int \sqrt{x^2+4x+5}\,dx$ *[6]*

Quiz #16. Friday, 1 March, 2002. *[25 minutes]*

1. Evaluate the following integral:

$$\int \frac{x^2-2x-6}{\left(x^2+2x+5\right)(x-1)}\,dx$$

Quiz #17. Friday, 8 March, 2002. *[25 minutes]*

1. Evaluate the following integral:

$$\int_2^\infty \frac{1}{x(x-1)^2}\,dx$$

Quiz #18. Friday, 15 March, 2002. *[18 minutes]*

Determine whether each of the following series converges or diverges.

1. $\displaystyle\sum_{n=0}^{\infty} \left[\frac{1}{n+1} + \frac{3^n}{3^n+1}\right]$ *[4]*
2. $\displaystyle\sum_{n=0}^{\infty} \frac{253}{3^n+1}$ *[6]*

Bonus Quiz. Monday, 18 March, 2002. *[15 minutes]*

Compute any *two* of 1–3.

1. $\displaystyle\lim_{t\to\infty} te^{-t}$ *[5]* 2. $\displaystyle\int_0^\infty te^{-t}\,dt$ *[5]* 3. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2+3n+2}$ *[5]*

Quiz #19. Friday, 22 March, 2002. *[20 minutes]*

Determine whether each of the following series converges or diverges.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^n}$ *[5]* 2. $\displaystyle\sum_{n=0}^{\infty} \frac{4n+12}{n^2+6n+13}$ *[5]*

Quiz #19. *(Alternate version.)* Friday, 22 March, 2002. *[20 minutes]*

Consider the series

$$\sum_{n=0}^{\infty} \frac{\arctan(n+1)}{n^2+2n+2}$$

Determine whether this series converges or diverges using:

1. The Comparison Test. *[5]*
2. The Integral Test. *[5]*

Quiz #20. Tuesday, 2 April, 2002. *[10 minutes]*

1. Determine whether the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n+\cos(n\pi)}{n+1}$ converges absolutely, converges conditionally, or diverges. *[10]*

Quiz #20. *(Alternate version.)* Tuesday, 2 April, 2002. *[10 minutes]*

1. Determine whether the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}+\sin\left(n\pi+\frac{\pi}{2}\right)}{n+1}$ converges absolutely, converges conditionally, or diverges. *[10]*

Quiz #21. Friday, 5 April, 2002. *[10 minutes]*

1. Find a power series which, when it converges, equals $f(x) = \dfrac{3x^2}{(1-x^3)^2}$. *[10]*

Quiz #21. *(Alternate version.)* Friday, 5 April, 2002. *[10 minutes]*

1. Find a function which is equal to the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n!}$ (when the series converges). *[10]*