

TRENT UNIVERSITY, WINTER 2018

MATH-CCTH 1080H Test #2

~~Tuesday, 13~~ Friday, 16 March*Time: 60 minutes*

Name: _____ Solutions _____

STUDENT NUMBER: _____ 0123456 _____

Question	Mark
1	_____
2	_____
3	_____
Total	_____ /30

Instructions

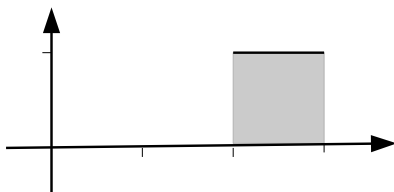
- *Show all your work.* Legibly, please!
- *If you have a question, ask it!*
- Use the back sides of the test sheets for rough work or extra space.
- You may use a calculator and an aid sheet.

1. Do *any two* (2) of **a-c**. [10 = 2 × 5 each]
- a. A non-standard fair four-sided die has faces numbered 0, 1, 3, and 6. What is the expected value of the number that comes up on a single roll of this die?
- b. The continuous random variable X has $f(x) = \begin{cases} 1 & 2 \leq x \leq 3 \\ 0 & x < 2 \text{ or } x > 3 \end{cases}$ as its probability density function. What is the expected value of X ?
- c. A fair coin is tossed twice and the random variable Y counts the number of heads that come up. Compute the expected value $E(Y)$ and variance $V(Y)$ of Y .

SOLUTIONS. **a.** Since the die is fair and four-sided, each face an equal chance of $\frac{1}{4} = 0.25$ of coming up on any given roll. Let F be the random variable that gives the number on the face that comes up. Then, by definition, the expected value of F is:

$$E(F) = \sum_f f \cdot P(F = f) = 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{4} + 3 \cdot \frac{1}{4} + 6 \cdot \frac{1}{4} = \frac{10}{4} = \frac{5}{2} = 2.5 \quad \square$$

- b. The graph of $f(x)$ is symmetric about $x = 2.5$:



It follows that $E(X) = 2.5$. \square

- c. Since the coin is fair, $P(H) = P(T) = \frac{1}{2}$ on any single toss. The possible values of Y are 0, 1, and 2, and their probabilities are:

$$P(Y = 0) = P(TT) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$P(Y = 1) = P(HT) + P(TH) = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$P(Y = 2) = P(HH) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

It follows that the expected value of Y is given by:

$$E(Y) = \sum_{y=0}^2 y \cdot P(Y = y) = 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} = 0 + \frac{1}{2} + \frac{1}{2} = 1 \quad \blacksquare$$

2. Do *all three* (3) of **a–c**.

Consider the following data: 2, 2, 3, 5, 6, 7, 7, 7, 8, 9

- Find the mean, median, and mode of the given data. [3]
- Find the standard deviation of the given data. [4]
- Suppose the data gives the number of heads that came up in ten different sets of ten tosses each of a coin. Is the coin biased or not? Explain why or why not as fully as you can. [3]

SOLUTIONS. **a.** There are ten data points, which is an even number.

The mean is just the average of the data:

$$\frac{2 + 2 + 3 + 5 + 6 + 7 + 7 + 7 + 8 + 9}{10} = \frac{56}{10} = \frac{28}{5} = 5.6$$

As there is an even number of data points, the median is the average of the middle two data points: $\frac{6 + 7}{2} = \frac{13}{2} = 6.5$.

The mode is the most common value among the data: 7, which occurs three times. \square

b. We first compute the variance V of the data, which is the average of the squares of the data points minus the square of the mean.

$$\begin{aligned} V &= \frac{2^2 + 2^2 + 3^2 + 5^2 + 6^2 + 7^2 + 7^2 + 7^2 + 8^2 + 9^2}{10} - (5.6)^2 \\ &= \frac{4 + 4 + 9 + 25 + 36 + 49 + 49 + 49 + 64 + 81}{10} - 31.36 \\ &= \frac{370}{10} - 31.36 = 37 - 31.36 = 5.64 \end{aligned}$$

The standard deviation σ is the square root of the variance: $\sigma = \sqrt{V} = \sqrt{5.76} \approx 2.37$. \square

c. It might be biased, but it's likely to be close to fair even if it has some bias. Note that if the ten data points give the number of heads that came up in each of ten sets of ten tosses of a coin, then their sum represents the number of heads in one hundred tosses of the coin. Fifty six heads out of a hundred tosses is not too unlikely for a fair coin, or one with a small bias. \blacksquare

3. Do *one* (1) of **a** or **b**. [10]

- a.** The graph on the right tracks the annual revenue [money coming in] of Company, Inc. It is the only data presented to the shareholders at the February 2018 annual meeting. Is there anything the shareholders should be concerned about?



- b.** If you were to pick an answer to this question at random from among the choices below, what is the probability that it would be correct? Explain your answer!

(1) 0.2 (2) $1/5$ (3) 0% (4) $4/10$

SOLUTIONS. **a.** There are two reasons the shareholders ought to be concerned. First, the given graph is a little deceptive in the scaling of the vertical axis. The difference of 200,000 in revenue between 2015 and 2016, for example, looks much less impressive in proportion to the totals of over 10,000,000 in revenue in both years. The second reason is that this graph is the only data being presented: to properly assess the state of a business you would also need the corresponding information on expenses and taxes, for two obvious items. The lack of relevant information, along with the deceptive presentation of what little is given, suggests that something is wrong at Company, Inc. \square

b. *Abandon all hope, ye who answered this! There is no right answer:* Choosing among the four choices at random, you would have a probability of $1/4 = 0.25$ of choosing each one. Since choices (1) and (2) are 0.2 and $1/5$, respectively, this value has a probability of $2/4 = 0.5 \neq 0.2 = 1/5$ of being chosen, so cannot be correct. Choice (4), $4/10$, cannot be correct, since it has a $1/4 = 0.25 \neq 4/10$ chance of being chosen. tion of the five alternatives must be an integer multiple of $1/5$, and $1/\pi$ is not an integer multiple of $1/5$. You might think this leaves a probability of 0 of selecting a correct alternative, but 0 is alternative (3). which has a probability of $1/4 = 0.25 \neq 0$ of being chosen at random . . . *The correct solution here is that there is no way to answer the given question, because it has been set up to be internally contradictory.* \square

[Total = 30]