MATH-CCTH 1080H – Mathematics for Everyday Life
TRENT UNIVERSITY, Winter 2018 in Peterborough

Solutions to Assignment #1
The Rule of Three

Cocker's Arithmetick was a standard basic math text that saw widespread use in England for about a century and a half, from the late Seventeenth to the early Nineteenth, going through over a hundred editions. Read the attached excerpt from the 22nd edition of this book and answer the following questions.

1. Explain the *Single Rule of Three Direct* presented by Cocker in your own words. What does the Rule accomplish? *[3]*

SOLUTION. The Single Rule of Three Direct specifies what to do if you want to find a number which is in a given ratio to a given number, the given ratio itself being specified by two numbers. That is, given numbers a, b, and d, the Rule tells you how to find c such that c is to d as a is to b (*i.e.* such that $\frac{a}{b}. = \frac{c}{d}$).

The actual algorithm given by the rule is to multiply a and d, and then divide the product by b. That is $c = \frac{a \times d}{b}$. ■

2. Fill in the blank in each of the following. If the result is not an integer, please give it in decimal form. *[5 = 5 × 1 each]*
a. 5 is to 3 as __ is to 87. **b.** 480 is to 132 as __ is to 11. **c.** 18.75 is to 3 as __ is to 4.
d. 15.3 is to 1.2 as __ is to 0.4. **e.** 59 is to 2.5 as __ is to 3.

SOLUTIONS. **a.** $\frac{5 \times 87}{3} = \frac{435}{3} = 145$

b. $\frac{480 \times 11}{132} = \frac{5280}{132} = 40$

c. $\frac{18.75 \times 4}{3} = \frac{75}{3} = 25$

d. $\frac{15.3 \times 0.4}{1.2} = \frac{6.12}{0.4} = 5.1$

e. $\frac{59 \times 3}{2.5} = \frac{177}{2.5} = 70.8$ ■

3. Why might Cocker have presented the *Single Rule of Three Direct* as he did, without using algebra? *[2]*

SOLUTION. Algebraic notation in something like the form we know it hadn't really trickled down to basic math education yet in the late 17th Century. (It was, to be sure, in use among professional mathematicians, physicists, and astronomers.) Without algebraic notation, algebra as we know it is next to impossible to present and use. The rhetorical/algorithmic way of presenting arithmetic and "algebra" used by Cocker is very much in the tradition used since Ancient Egypt and Mesopotamia, allowing for changes in languages, writing, and number systems. ■

REFERENCE

1. *Cocker's Arithmetick, perused by J. Hawkins* (22nd Edition), by Edward Cocker & John Hawkins, London, 1702. May be read at or downloaded in pdf form from Google Books: `books.google.ca/books?id=GWcFAAAAQAAJ`

Also if 3, 9, 21, 63, were given (which are interrupted) I say 9 times 21 is equal to 3 times 63, which is equal to 189.

From hence ariseth that precious Gem in Arithmetick, which for the Excellency thereof is called the *Golden Rule*, or *Rule of Three*.

CHAP. X.

The Single Rule of Three Direct.

1. THE *Rule of Three* (not undeservedly call'd the *Golden Rule*) is, that by which we find out a fourth number, in proportion unto three given Numbers, so as this *fourth Number* sought may bear the same Rate, Reason, or Proportion to the *third* (given) *number*, as this second doth to the first, from whence it is also called the Rule of *Proportion*.

2. Four *Numbers* are said to be *Proportional*, when the first containeth or is contained by the second, as often as the third containeth or is contained by the fourth. *Vide* Wingate's *Arith Chap.* 8. *Sect.* 4.

So these *Numbers* are said to be *Proportionals, viz.* 3, 6, 9, 18, for as often as the first *Number* is contained in the second, so often is the third contained in the fourth, *viz.* twice. Also 9, 3, 15, 5, are said to be *proportional*, for as often as the first *Number* containeth the second, so often the third *Number* containeth the fourth; *viz.* 3 times.

3. The *Rule of Three* is either simple or composed.

4. The simple (or single) *Rule of Three*, consisteth of 4 *Numbers*; that is to say, it hath 3 *Numbers* given to find out a fourth; and this is either Direct, or Inverse. *Vide Alsted. Math. lib.* 2. *cap.* 13.

5. The single *Rule of Three Direct*, is when the proportion of the first *Term* is to the second, as the third is to the fourth; or when it is required that the *Number* sought

(*viz.*) the fourth Number must have the same proportion to the second, as the third hath to the first.

6. In the *Rule of Three*, the greatest difficulty is (after the Question is propounded) to discover the order of the 3 Terms, *viz.* which is the first, which is the second, and which the third, which that you may understand, observe, That (of the three given numbers) two are always of one kind, and the other is of the same kind with the proportional number that is sought; as in this Question, *viz.* If 4 yards of Cloth cost 12 shillings, what will 6 yards cost at that rate? Here the two numbers of one kind are 4 and 6, *viz.* they both signifie so many yards; and 12 shillings is the same kind with the number sought, for the price of 6 yards is sought.

Again, observe, that of the 3 given numbers, those two that are of the same kind, one of them must be the first and the other the third, and that which is of the same kind with the number sought, must be the second number in the *Rule of Three*; and that you may know which of the said numbers to make your first, and which your third, know this, that to one of those two numbers there is always affixed a demand, and that number upon which the demand lieth must always be reckoned the third number. As in the forementioned Question, the demand is affixed to the number 6, for it is demanded what 6 yards will cost? and therefore 6 must be the third number, and 4 (which is of the same denomination (or kind) with it) must be the first, and consequently the number 12 must be the second, and then the numbers being placed in the forementioned order, will stand as followeth, *viz.*

yards	s.	yards
4	12	6

7. In the Rule of *Three Direct* (having placed the numbers as is before directed) the next thing to be done will be to find out the fourth number in proportion, which (that you may do), multiply the second number by

the third, and divide the product thereof by the first, (or which is all one) multiply the third term (or number) by the second, and divide the product thereof by the first, and the Quotient thence arising is the 4th number in a direct proportion, and is the number sought, or Answer to the question, and is of the same denomination that the second number is of. As thus, let the same Question be again repeated, *viz.* If 4 yards of Cloth cost 12 Shillings, what will 6 yards cost?

Having placed my *numbers* according to the sixth Rule (of this Chapter) foregoing, I multiply (the second number) 12 by (the third number) 6, and the product is 72, which product I divide by (the first number) 4, and the quotient thence arising is 18, which is the fourth proportional or number sought, *viz.* 18 shillings, (because the second number is shillings) which is the price of the 6 yards, as was required by the question. See the Work following:

```
   yds    s.   yds    s.
If 4 —— 12 —— 6 —— 18
         6
       ----
    4) 72 (18 shillings
       ..
        4
       ----
       32
       32
       ----
       (0)
```

Quest. 2. Another Question may be this, *viz.* If 7 *C.* of Pepper cost 21 *l.* how much will 16 *C.* cost at that rate?

To resolve which question, I consider that (according to the 6th Rule of this Chapter) the terms or numbers ought to be placed thus, *viz.* the Demand lying upon 16 *C.* it must be the third number, and that of the same kind with it must be the first, *viz.* 7 *C.* and 21 *l.* (being of the same kind with the number sought) must be the second number in this question; then I proceed according