

Mathematics 1001H – Precalculus Mathematics

TRENT UNIVERSITY, Summer 2016

Assignment #3

Some Significant Sequence Sums

Due on Tuesday, 31 May, 2016.

In all that follows, n is some arbitrary positive integer.

1. Explain why $1 + 2 + 3 + \cdots + (n - 1) + n = \frac{n(n + 1)}{2}$. [2]

NOTE: We did this one in passing in class one day, if memory serves ...

2. Suppose a and d are numbers. Find a formula (in terms of a , d , and n) for the sum $a + (a + d) + (a + 2d) + \cdots + (a + nd)$. [2]

NOTE: The sequence of numbers $a, a + d, a + 2d, \dots$, is the *arithmetic sequence* with first term a and *common difference* d . Their sum is the corresponding *arithmetic series*.

3. Suppose a and $r > 0$ are numbers. Find a formula (in terms of a , r , and n) for the sum $a + ar + ar^2 + ar^3 + \cdots + ar^n$. [3]

Hint: Try multiplying the sum by $1 - r$ and see what happens.

NOTE: The sequence of numbers a, ar, ar^2, \dots , is the *geometric sequence* with first term a and *common ratio* r . Their sum is the corresponding *geometric series*.

4. Suppose a , d , and $r > 0$ are numbers. Let $a_0 = 0$ and, for each $k \geq 0$, let $a_{k+1} = a_k r + d$. (Thus $a_1 = ar + d$, $a_2 = (ar + d)r + d$, $a_3 = [(ar + d)r + d]r + d$, and so on.) Find a formula (in terms of a , d , r , and n) for the sum $a_0 + a_1 + a_2 + \cdots + a_n$. [3]

Hint: Use algebra to take it all apart and reassemble it in another way. Keep your answers to the previous questions in mind, too.

NOTE: There's a name for this, but – darn it! – I can't remember it ...