MC^2 : Mathematics Classroom Collaborator

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Abstract

Relative to other disciplines, post-secondary mathematics and statistics courses typically have little in-class interaction and discussion, due, at least in part, to a number of barriers that make communication difficult. In this paper we introduce a messenger-like interface for mathematics, and we describe the design features that make its use as easy as possible for those who have no experience communicating mathematics online.

1 Introduction

Although vast quantities of academic knowledge have existed in libraries for thousands of years, learning has most often been rooted in the lecture. And with internet sites like YouTube, a learner can now easily access a captivating lecture on almost any topic. Yet, universities are perhaps more important than ever before.

What makes the university learning experience so valuable, in part, are the opportunities it provides to discuss, to debate, and to pose questions to the instructor. In a course on literature, for example, small group seminars may facilitate dialogue, while in the experimental sciences, labs may provide opportunities for groups of students to work together to solidify their knowledge, as a lab demonstrator provides feedback or assistance.

Mathematics classrooms, by contrast – particularly, in the large first-year service courses – may be the among least-engaged places on campus. More than in perhaps any other subject, instruction involves "chalk and talk" – the instructor writes math on a chalkboard, and students copy it down onto paper. Students rarely ask questions, and even seminars and tutorials typically involve a teaching assistant working through examples, and students diligently copying those examples into their notes.

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In: A. Kohlhase, M. Pollanen (eds.): Proceedings of the MathUI'17 Workshop, Edinburgh, Scotland, 17-07-2017, published at http://ceur-ws.org

Many instructors have tried to address this widely-recognized problem with, for example, "flipped classrooms" and immediate feedback systems (i.e., "clickers"). The problem is a difficult one to remedy, however, for it is rooted in a number of external factors that make students less likely to engage in quantitative courses. These factors include:

Math Phobia: In mathematics and statistics *service* courses, the majority of students feel anxiety. It has been found that 85 percent of students have math anxiety in a first-year mathematics course [Per04]. And even in graduate studies, where one might expect students to be more inclined to embrace a challenge, 80 percent of students in the social and behavioural sciences were found to suffer from statistics anxiety [Onw03]. It may be reasonable, then, to presume that anxiety towards the subject could make students less likely to engage in an in-class dialogue.

Dualism: Every instructor tells his/her students that "there are no bad questions". This, however, is not how mathematics is perceived. In a literature course, when asked about the motives of a character, a student may respond, modify, and clarify their answer until it meets with approval, perhaps even if the final answer is diametrically opposed to what they originally stated. Mathematics, by contrast, is seen as something that is either right or wrong. Because, psychologically, no one wants to be wrong, even students who are confident in their math abilities may be reluctant to answer questions.

Classroom Interface: Mathematics is meant to be written, not just spoken. When an instructor poses or answers a question, he/she can use chalkboard or whiteboard as an aid. But when a student asks or answers a question, their only option is to verbally express the mathematics. This puts students at a disadvantage.

Lack of Democracy: The classroom is not very democratic, as there are groups within the classroom who don't have an equal voice. It has long been known that women might not ask a proportionate number of questions in math classes [Kru85], and anecdotal reports suggests that international students may be shy about asking questions in their non-native language. This pattern would be consistent with other sciences. In biology, for example, it has been reported that both male and female faculty members are more likely to call on males to answer questions in class [Gru16]. And in computer science, it has been found [San15] that female undergraduate students ask 37 percent fewer questions than their male peers – a gap that persists at the graduate level, where females were found to ask 42 percent fewer questions than their male peers.

In previous work [Hoo06], we have studied the use of online technology for office hours. Traditional office hours are usually not well-attended by students, often with fewer than 10 percent of students seeing their instructor outside of the classroom. We found, however, that in a first-year mathematics service course, 40 percent of students in the class might attend a single *online* office hour. In subsequent surveys, students frequently cited the anonymity and their fears and anxieties as a reason for attending online office hours. Likewise, in computer science, [Jon13] showed that an anonymous discussion tool can increase participation and learning outcomes. While neither study addressed gender, [San15] found that with the online bulletin board tool Piazza, female STEM students were more likely to use the anonymity feature than their male counterparts.

It may, therefore, be beneficial to have a tool in mathematics that allows students to easily and anonymously ask and answer questions, or to collaborate with small groups of students in a large-class environment. Ideally, this tool should make the entry of mathematics as easy and as intuitive as possible, include the option for anonymity, and work on a variety of platforms – smartphones, tablets, and notebook computers. In the sections that follow, we will discuss the challenges of mathematical entry; describe the user requirements for such an interface; and finally, introduce a prototype of such an interface – *Mathematics Classroom Collaborator* (MC^2) – and discuss its in-class potential.

2 Challenges in Mathematical Input

Entry methods for mathematical expressions have changed little in the decades since they were first designed to create documents on a personal computer by users with a high degree of mathematical expertise. What has changed, however, is the need for novices to interact with mathematics electronically. In post-secondary education, students must be able to interact with relatively sophisticated mathematical equations. A first-year psychology student, for example, usually needs to take an introductory course in statistics, where they would be introduced to the formula for the bivariate correlation coefficient:

$$r_{xy} = \frac{n \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i \right)^2} \sqrt{n \sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i \right)^2}}$$

A mathematician might write this equation using TEX, which could represented as

```
r_{xy} = \frac{n \sum_{i=1}^{n} x_{i} y_{i} - \sum_{i=1}^{n} x_{i}
\sum_{i=1}^{n} y_{i} }{ \sqrt{n \sum_{i=1}^{n} x_{i}^{2} - \left( \sum_{i=1}^{n} x_{i}
\right)^{2} } \sqrt{n \sum_{i=1}^{n}  y_{i}^{2} - \left( \sum_{i=1}^{n} y_{i} \right)^{2} } }
```

For a non-math major, this is likely incomprehensible, due to the complexity of mathematical notation. As seen from the equation, mathematical writing, unlike text, has a two-dimensional layout, and also contains many symbols not found on a keyboard. An alternative to the text-based approach found in many word processors, such as Microsoft Word, is a structure-based editor, where mathematical structures and symbols are separately entered from palettes of buttons (see Figure 1). Both the TEX and structure-based approach present difficulties for novice users in that they force the writer to pre-parse their expression and often write it in a different order than they would on paper. This may not only be non-intuitive and have a steep learning curve – it may also increase the cognitive load of the user and interfere with the task of communication if used in a collaborative environment.

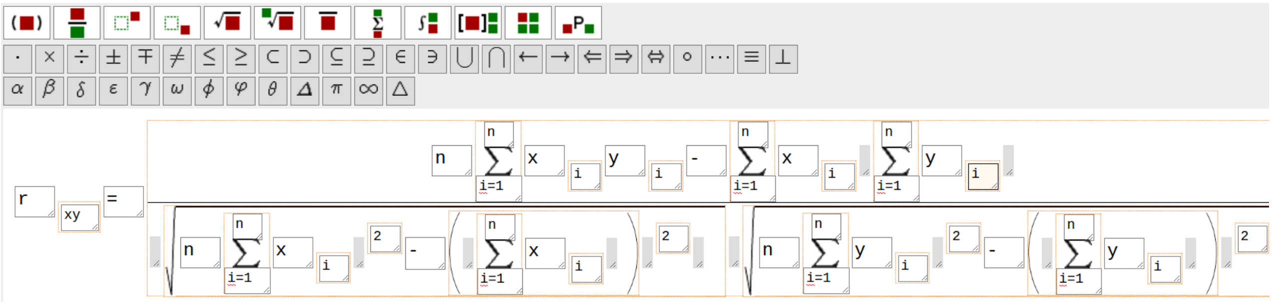

Figure 1: Bivariate correlation coefficient equation in a structure-based editor.

The pen-and-paper is a powerful and familiar paradigm in mathematical writing. At present, however, pen-based input is not robust enough for a collaborative environment. Handwriting recognition rates are low, and few devices are equipped by default with a stylus. A further problem is that while pen-input is familiar to all users, the pen-and-paper model does not extend to correcting and modifying expressions when they have already been recognized and converted into typeset notation. It is not clear at this time how to make formula manipulation as natural and intuitive as writing with a pen on paper [Pol17].

3 Interface Design

Our aim is to develop a mathematics communication and collaboration environment for a classroom. Thus, it must work on a variety of different hardware platforms – tablets, smartphones, and laptop computers – running on a variety of different operating systems, including Windows, MacOS, iOS, and Android. Ideally, students should have easy access to the software, without having to pre-install anything, so a browser-based architecture makes sense.

In terms of input, the methods include keyboard, mouse, stylus, touch screen, and camera. The use of voice assisted input is just in its infancy in mathematics [Guy04], and it is difficult to envisage how it might be useful in a mathematics classroom environment, so voice was not considered as an option in our application.

In recent years, a myriad of mobile applications that mimic the sending of SMS messages have been developed (e.g., WhatsApp, Google Hangouts). With the exception of allowing for the sending of images, voice notes, and inserting emojis/stickers, however, these applications are text-based and don't allow for the creation of rich symbolic content, such as mathematics. The few applications for mathematical communication use TEX-based input (e.g., MathIM) or follow a shared whiteboard format [Pol06]. The whiteboard communication paradigm has a few drawbacks, relative to the text-messaging interface model, for our application. In particular, although common, many users have not seen or used a shared-whiteboard application before. But virtually everyone with a smartphone has sent a text message, so it can be assumed there is universal familiarity with the basic use of the interface. This is important, for there may be a significant barrier to use by students if they feel they could potentially be humiliated in front of classmates by incorrectly sending their first message in a communication medium they are not familiar with. A further advantage of the messaging application is that while the conversation could be in real-time, it could also evolve over a period of hours or days, and users can scroll up to view the conversation history. Whiteboard interfaces are better suited to viewing real-time content being created, for there is not a standard way for users to review the conversation history if they are arriving late. Moreover, the small screen-size of smartphones makes whiteboards harder to implement on such devices. For these reasons MC^2 was designed around the design of a texting application.

One major difference between MC^2 and messaging applications is that the latter almost always need to be installed on the smartphone, while MC^2 has been designed as a Web-application. This makes it easier for MC^2 to support a wide variety of platforms, such as notebooks (PC, MacOS), tablets, and smartphones (Android, iOS), and it avoids the associated application stores (AppStore, PlayStore). This could also make it easier for the mathematical community to adapt it for their own use, and seamlessly incorporate it into learning management systems. A Web-based application might also provide a lower barrier for students to try it for the first time, and ease privacy concerns about what data the application can gather from the phone. The client-side of MC^2 is written in JavaScript and utilizes SVG and MathJax, while the server-side is also JavaScript and utilizes node.js, making it easy to install, as it contains an integrated Web server.

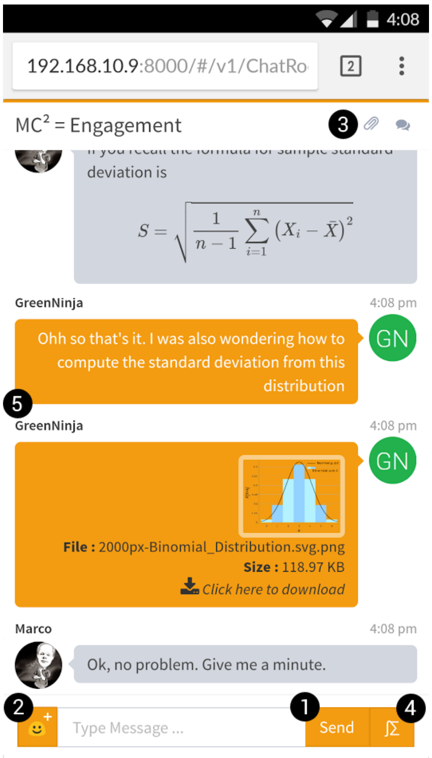

Figure 2: MC^2 Chat Screen — (1) Text input field. (2) Insert emoticons, TEX, or symbols into text field. (3) Input image from camera or gallery. (4) Launch the mathematical expression editor. (5) Dialogue Pane: clicking on an image or mathematical expression launches the expression editor for annotating or modification.

On the surface, MC^2 presents itself as a familiar texting application with an interface similar to Telegram, Google Hangout, or WhatsApp, with the mathematical capabilities mapped to common interactions with these applications to ease the learning curve (see Figure 2). In what follows, we highlight some of the differences.

① **Text Field:** In addition to typing regular text, the text field supports the insertion of TEX and Unicode math symbols. On a computer, for users adept at TEX this can make mathematical entry quick. Of course, for smartphone users, this is much more challenging. In general, entry of even regular text on smartphone keyboards is cumbersome, and users have acquired a great deal of practice with their system keyboard. So, for text entry we use the default native keyboard.

② **Emoticons/TEX/Math Symbols:** TEX input requires a number of symbols such as `$`, `\`, `{`, `}`, `_` , and `^`. On smartphone keyboards, these symbols are often difficult to access quickly. In MC^2, these symbols – some simple templates of TEX expressions that can be inserted into the text field – as well as some common unicode math symbols can found under the emoticon/symbol button. A basic assortment of emoticons is also included to facilitate expression in the conversation.

③ **Camera/Image:** The camera/image button allows a user to upload an image. This could allow a user to write mathematics on a piece of paper and upload it, which could be convenient for those accessing the chat from a smartphone. This method has a number of possible drawbacks. Those using a laptop instead of a smartphone might find taking a picture of an equation difficult. Even for those students with smartphones, some may feel shy about taking a photo if their phone has a visual/audible indicator when a picture is taken, and users outside the classroom might not have easy access to a pen and paper if, for example, they riding on a crowded bus. Furthermore, photographs of mathematical equations are difficult to reuse, manipulate, and share with other applications in a meaningful manner. In the future, however, as recognition algorithms for mathematics from images improve, it may be possible to convert these images into mathematics. We anticipate that the image upload feature will be used to upload screenshots of questions from homework or online lecture notes.

④ **Mathematical Editor:** Complex mathematical expressions can be entered by accessing the mathematical editor (see Figure 3), where equations can be input using a diagrammatic equation editor. If a user has access to a stylus, equations can also be hand drawn. On a notebook, the mathematical editor and messenger load side-by-side. On a smartphone with limited screen real estate, however, the two views must be toggled, in a way similar to how a user in a texting app views a full-sized image that has been received. The diagrammatic editor allows for the free-form entry of mathematical expressions, allowing users to place mathematical symbols, which can be selected from menus or from the keyboard, anywhere they wish on the writing surface. The symbols can be selected, moved, and resized based on diagram editor UI principles as developed in [Pol14a] and [Pol14b]. This UI paradigm works well not only with the keyboard/mouse interactions of a computer, but also with the drag-and-drop actions on a smartphone or tablet. Using this diagrammatic form of interface, it was shown that students were able to write expressions more quickly than when using a structure-based editor, and also, more importantly, in the same order as they would on a piece of paper [Goz09]. Diagrams can also be drawn and can, along with mathematical expressions, be inserted into the messenger, or completed mathematical expressions can be recognized using a baseline structured approach ([Zan02], [Pol07]) and converted to TEX which can be rendered and displayed by the messenger. Currently, only expressions created with the diagrammatic drawing tools can be converted to TEX. And if one uses a stylus to handwrite mathematics, this can be inserted into the chat as an image, though, in the future, algorithms for handwriting recognition could be considered.

Because there are hundreds of math symbols that one might need to access, and also because the native keyboard keyboard on a smartphone combined with the Web-browser URL header can cover up to two-thirds of the screen space, a custom keyboard was implemented. The custom keyboard was implemented as a translucent layer, allowing the expression to be seen while it is being entered.

⑤ **Dialogue Pane:** The dialogue pane is where the conversation is displayed in a chronological order. In use, it is nearly identical to the panes found in text messaging apps. In those applications, when a user clicks on a picture, it is enlarged for closer viewing and optionally shared with other apps. In a similar fashion, when an equation or image is clicked on by an MC^2 user, it is enlarged and loaded into the mathematical editor, where the equation can be edited and inserted into the end of the conversation. Alternatively, if the image is a photograph or other image it can be marked up and annotated with digital ink and inserted into the conversation. One additional feature of the Dialogue Pane is that it automatically renders messages with embedded TEX through MathJAX.

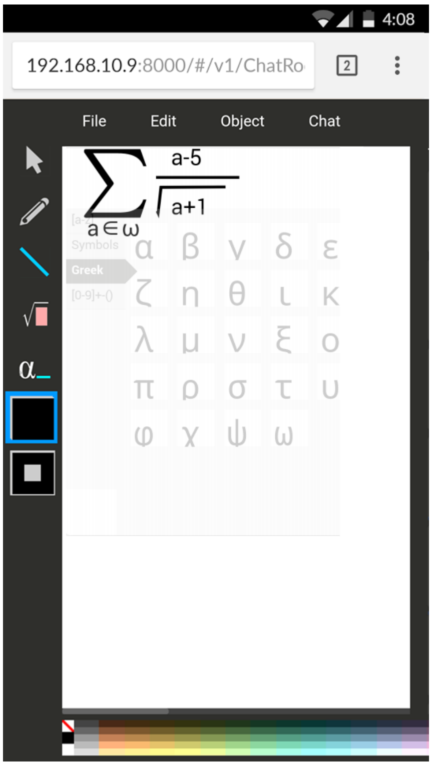

Figure 3: MC^2 Mathematical Editor screen with translucent symbol keyboard displayed.

4 Potential New Applications

Although the use of communication technology in mathematics is not widespread, we have found in previous work that a large increase in office hour attendance occurs when online office hours are made available [Hoo06]. Other studies have shown that, in general, the overwhelming number of students would like to see more digital communication options [Hel12]. As we already implemented online office hours using a whiteboard technology [Pol06], our primary reason for creating MC^2 is to create a more engaged classroom. For this we envisage a number of possible scenarios:

Many to One: The instructor could open a discussion at the beginning of class. This would allow students to send questions to the instructor during class to seek clarification during the lecture. The instructor could periodically check those questions as the class progresses, so as to address any concerns. This could also be helpful for distance learners, if the lecture is simulcast online.

One to Many: The instructor could send a question to the class to work on. He/she could, for example, pose a question and provide a prize to the first student/group that posts a correct solution to the chat. Perhaps in a future version, the recognition technology could be used to automatically check an answer for correctness. This could be useful in replacing immediate feedback systems (i.e., "clickers") used in large classes. These systems typically allow an instructor to post a multiple-choice question, and have students use their clicker device to answer. But instead of multiple choice, perhaps through an extension of MC^2, students might be able to answer the question by entering a correct expression.

Many to Many: Large lecture halls often offer theatre-style seating and do not allow students to work in small groups, in spite of the fact that group-based discussion has been shown to have many pedagogical benefits [Maz09]. An in-class discussion tool could allow for members of small groups to interact in a large lecture-hall setting. Alternatively, a class discussion could be running in parallel to the lecture that allows students to ask for clarification from other students or teaching assistants.

5 Conclusion

Increasing student communication, collaboration, and engagement, both inside and outside the classroom, is an important aim for many instructors. One key to achieving this goal is to utilize communication technology. However, the development of robust communication apps in mathematics has been slow. One problem is that because mathematical input methods have largely been designed for expert users, they have a steep learning curve. Moreover, they have been largely designed with document creation in mind, not for communication where expressions which have been written may also need to be easily modified. Still further, mathematical input systems have largely been designed around the mouse and keyboard hardware paradigm, rather than the touchscreen found on smartphones.

In this paper, we have introduced a Web-based application called MC^2: the *Mathematics Classroom Collaborator*. The application is designed to make mathematical communication as intuitive as possible, and to flatten the learning curve, by utilizing other UI models already known to most users, such as text-messaging apps and diagram-editors.

Traditionally, mathematics and statistics courses have lagged behind other disciplines in the levels of student engagement. And with advances in technology, many non-mathematics courses have adopted new communication tools [Hel12] and social media [Ayd12], leaving the mathematical science to fall even further behind. From our experience with anonymous online office hours, however, we believe that students do want to communicate more with their instructor and expect peer-to-peer interactions. Thus, creating a tool that technically enables mathematical communication would be a first step forward. Through the launch of MC^2 in a classroom setting in courses in the mathematical sciences, and the exploration of new models of classroom interaction, the engagement gap between mathematics and other disciplines may close, which may lead to better learning outcomes and student retention.

Acknowledgements

This work was supported by a research grant from *eCampus Ontario*.

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