

An Efficient Gamified Model for Online Homework in the Mathematical Sciences

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ABSTRACT

In this paper we develop an online homework model for post-secondary mathematics and statistics service courses. The model utilizes gamification and the testing effect to increase engagement of users in the learning process. Unlike many environments employing game-based learning that make use of complex game-based elements such as story-lines or avatars, our model is comprised of game elements that largely involve the sequencing of questions and grades for retry attempts and that are, therefore, essentially transparent. It is thus efficient for students in that all of their effort is devoted to answering questions, rather than to gameplay. The model is general, and because it can easily be reused with different content, it is efficient in terms of its usage of instructor and developer resources. We analyze the effectiveness of the model using both qualitative and quantitative survey results

Keyword: Online Homework, Gamification, Mathematics Education

1. Introduction

Post-secondary students in the sciences and social sciences are typically required to take one or more introductory mathematics or statistics courses. These students differ greatly in terms of their preparation — and even for those with relatively strong backgrounds, mathematics is a subject that causes anxiety. As a result, levels of student engagement and retention rates in mathematics service courses can be lower than those found in other disciplines.

In this paper, we introduce an online instructional model that employs cognitive principles, such as the testing effect, in a gamified environment to allow students to

receive the individual level of practice that they require within a context that both engages and stimulates them.

The testing effect (McDaniel, Roediger, & McDermott, 2007), a robust discovery in educational psychology, demonstrates that students are most focused, in-tune and best able to learn new material while actually writing a test. This observation holds, regardless of the format of the test or the quality of the feedback provided. Thus, an argument can be made for much more frequent testing. One drawback of frequent testing, of course, is that it requires a lot of instructional resources to create and grade tests, especially for large classes. One common approach to overcoming these difficulties is to employ response systems (i.e., “clickers”). But even with clickers, the competition with instruction for scarce class time limits the number of test questions that can be asked.

Further work on the testing effect has shown that students are best able to correct misconceptions if they are able to do so during a test. Assessment-for-learning is an instructional model that leverages this observation by allowing students to have multiple attempts at answering questions (usually for diminishing marks). A version of this that has recently become popular involves the use of multiple-choice scratch cards that allow for students to make multiple attempts at a multiple choice test (Epstein, Epstein, & Brosvic, 2001).

Another area of research that has drawn a lot of attention in recent decades is that of game-based learning, which exploits the innate ability of people to learn from games and which provides an engaging learning environment. Full-fledged games do, however, have inefficiencies. They can be very expensive to develop, while perhaps being useful for only a single learning module in a course, and they may also require a large time-commitment from the learner to complete. The concept of gamification is one approach for addressing these challenges. With gamification, game-based elements (such as points, levels, and rewards) are added to learning environments to motivate students, circumventing the need to develop full-fledged games.

In this paper, we will develop an instructional model that makes a template-driven mathematics homework assignment both more test-like and more game-like. In the following sections, the literature on gamification in mathematics is considered, and elements of gamification are contrasted with some popular well-known games. We will then provide background material concerning template-driven mathematics homework environments and results from in-course use. Finally, our new online

homework model is introduced.

2. Gamification

Play is one of the main mechanisms by which children learn, and even older children and adults continue to enjoy playing games. Because this has been recognized by educators for over a century, game-based design has been incorporated into classroom learning. With the advent of personal computers, computer-based educational games have been developed. And along with these developments, scholars have been active in conducting research on game-based learning, yielding over a thousand research papers.

Full-fledged educational games can, however, be expensive to develop and time-consuming to play, so they may yield low knowledge transference per unit of development and user costs. This is particularly true in a post-secondary mathematics service course, where a different topic covered in each lecture might require a different game.

In recent years, educators have become more attuned to these limitations, and many have become intrigued by the concept of efficient gamification — the addition of game-based elements to educational content to engage students and improve learner motivation (Kapp, 2012). In the last year alone, there have been several noteworthy review articles on the area of gamification (Nah, Zeng, Telaprolu, Ayyappa, & Eschenbrenner, 2014) and (Dicheva, Dichev, Agre, Angelova, 2015).

Nah et al. (2014) reviewed the literature and found only eight design elements that had, thus far, been used for gamification:

1. Points: can measure achievement
2. Level/Stages: partitioning the game to create a sense of progression
3. Badges: marks of achievement or task accomplishment
4. Leaderboards: social recognition for top players
5. Prizes and Rewards: in-game rewards, such as character upgrades
6. Progress Bars: track overall progress
7. Storyline: wrapping the game in a narrative or story for the game
8. Feedback: frequent or immediate feedback

The learning outcome associated with gamification that was most frequently reported in the study was engagement, followed by motivation.

There has been no specific review study on gamification in mathematics. In (Goehle, 2013), however, two gamification elements — levels and badges — were incorporated into the popular open-source homework environment WebWorK.

3. Popular Classic Games

Many classic video games, such as *Tetris* and *Minesweeper*, require a player to solve simple monotonous puzzles. But the games are well-known for their addictive qualities. Many of the highest-rated game-shows in history, such as *Jeopardy!* And *Who Wants to be a Millionaire*, are contests of knowledge. Classic board games such as *Scrabble* and Chess are intellectual in nature. From a casual contemplation of these popular games, it may not be readily apparent how well they related to the list of gamification elements in the previous section. Since these games seem to be able to engage a large cross-section of the population in intellectual pursuits, they might provide some lessons for game-based educational design. Let us consider some of these games and the gamification elements that they include.

Tetris

Tetris is perhaps one of the most addictive, yet also the simplest, of all video games. Hundreds of variants of the *Tetris* concept exist. The basic idea is that users are required to place a sequence of falling shapes (tetrominoes) in such a way that complete rows are created (and then disappear). Players are able to rotate and laterally move a falling block. When the user is unable to complete a row, the blocks continue to pile up. Once the blocks reach the top, the game is over.

Given the extreme simplicity, and the fact that in most variations of *Tetris* the player can never win — the pieces just keep falling at increased speeds as milestones are reached — it is interesting to ask why users would find it to be so engaging. Most games, such as *Tetris*, benefit from a Sunk Cost Bias in which people consider sunk costs in making a decision. Many people would, for example, be less willing to walk out of a bad movie that they paid for than out of one that was free, for they will take into consideration the value of the ticket — a consideration that is irrational because the price cannot be recovered. With many games, this effect can be seen: the longer you have been playing, the more pressure you feel to see the current attempt to completion.

Another element employed by many successful games is risk (Pollanen, Cater, Kang,

2015). The fortunes of the player ebb and flow as the game progresses. In *Tetris*, this manifests itself in the user constantly being in a state of peril, only a few moves away from the blocks reaching the top. But even until the last second, with some luck and skill, there is always the possibility that the user may be able to form some complete rows and move further away from the brink of loss.



Figure 1: A variant of the popular game *Tetris*

It has been suggested that *Tetris* is addictive because of the Zeigarnik effect (Stafford, 2012) — a psychological observation that many waiters had fantastic memories for complex orders for a party, but then forgot it immediately after the order was served. Zeigarnik speculated that the human brain has a sticky memory for incomplete tasks.

Whether it is sunk costs, risk, or the Zeigarnik effect, successful, engaging games often increase the pressure or anxiety that a user experiences as the game continues.

Minesweeper

Minesweeper is a game in which a player attempts to clear a rectangular matrix with hidden mines. When a user clicks on an uncleared cell, what is beneath the cell is revealed. If there is a mine, the game is over. If the cell does not have a mine, that cell and a region of connected cells with no adjacent mines are cleared. Cells connected to the cleared region have a number in each indicating the number of adjacent cells that

have mines, thus providing a logic puzzle as the user clicks on uncleared cells hoping to uncover all non-mine cells without ever clicking on a mine.

Minesweeper originated in the 1960s on mainframe computers, but one of the most popular variants is that included with many versions of the Microsoft Windows operating system. While *Tetris* and *Minesweeper* both employ the concept of peril, in *Tetris* this is accomplished by having to make quick decisions, while in *Minesweeper* it is a slow contemplative peril in which any next step may end the game.

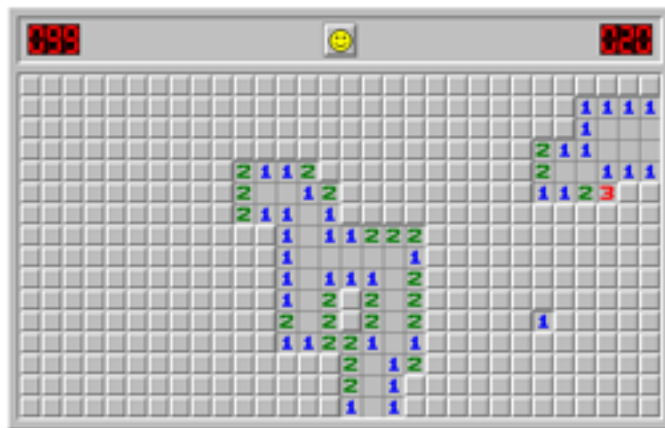


Figure 2: A variant of the popular game *Minesweeper*

Jeopardy!

Jeopardy!, one the most successful game-shows in television history, first aired in the United States in the 1960s. In the game, three contestants compete by selecting from 25 questions under five categories. Questions vary by dollar amount, and, once revealed, contestants must buzz in to be able to answer the question first. If answered correctly, the dollar amount of the question is added to the contestant's score and the contestant is then allowed to choose the next question. If one answers incorrectly, however, the dollar amount is deducted from the score. Thus, there are elements of time pressure and risk.

It has been speculated that what makes shows like *Jeopardy!* engaging is the Zeigarnik effect (Stafford, 2012). One version of this effect is that once a question has been asked, people have a need to know the answer. Another feature that keeps the game exciting is that, until its end, it is possible for any player to catch-up and win. This design is accomplished by incorporating a number of features. After the first set of 25 questions, there is a second round of 25 questions, called "Double Jeopardy", in which the prize values for the questions are double those of the previous round. In

addition, in each round there are two hidden “Daily Double” questions for which any amount can be wagered by a person selecting it. Furthermore, the game ends in a “Final Jeopardy” question in which each user attempts to answer the question and can wager any amount.

Who Wants to be a Millionaire

Who Wants to be a Millionaire, like *Jeopardy!*, is a popular knowledge-based game-show format with many international variations. In the American version, a player answers a series of increasingly difficult multiple choice questions up to a maximum prize amount of \$1 million. If the person answers a question incorrectly, the game ends. However, the player can walk away at any time and keep the money he or she has earned up to that point in the game. There also are safety-nets so that when a contestant reaches various levels that person will walk away with a guaranteed payout. In addition, a player also has three lifelines, each of which can be used once:

1. Fifty-Fifty: two incorrect responses are removed, so that the contestant is left with a choice between only two possible answers.
2. Ask the Audience: the contestant can poll the audience regarding their opinions as to which answer is correct.
3. Phone a Friend: the contestant may make a telephone call to a friend to ask his or her opinion regarding which response to select.

In considering the above popular games, it appears that there are other important gamification elements that have thus far not been explored in the education literature. These elements are as follows:

1. Risk: ebb and flow of the fortunes of the player.
2. Safety-Net: a level below which the player cannot fall.
3. Catch-Up: maintaining a hope that a player can meet the objective.
4. Sunk Cost: once a player has invested a lot of time in a game they do not want to quit.
5. Zeigarnik Effect: the need to complete the question or task.

4. Interactive Mathematics Assignments

Service courses in mathematics and statistics are often essentially skills-based. For instance, in an introductory calculus course most of the time is spent learning to apply rules to various functions. When given a particular function, students are expected to

find the derivative, the maximum value, or the area underneath the curve.

One advantage that mathematics has over other disciplines is that these types of questions could be generated by making use of templates. For example, very general template functions can be created, such as the following:

$$\frac{(a_1x^{b_1} + a_2x^{b_2} + a_3x^{b_3})^{c_1}}{(a_4x^{b_4} + a_5x^{b_5} + a_6x^{b_6})^{c_2}} d^{a_7x^{b_7} + a_8x^{b_8} + a_9x^{b_9}} \left[\ln(a_{10}x^{b_{10}} + a_{11}x^{b_{11}} + a_{12}x^{b_{12}}) \right]^{c_1},$$

which by randomly populating the parameters could represent a very large number of different functions:

$$(i) \frac{x^2 + 2x + 1}{x^2 + 1} \quad (ii) \sqrt{x^3 + x^2 + 1} \quad (iii) \frac{\ln x}{x^3 + 1} \quad (iv) \frac{x^4 e^{3x}}{\ln(x^2 + 1)}.$$

Another advantage of mathematics is that with the assistance of a computer algebra system, free-form answers from students can be scored. Thus, an unlimited number of practice problems could be generated and scored automatically, and even full solutions might be generated using templates. Answers do not have to be limited to algebraic input, but interactive templates, such as a curve sketching exercise are possible (see Figure 3). Templates for word problems for a course, such as discrete mathematics, could also be created. Consider the following example:

Question: How many ways can 10 identical books be placed in 5 unlabeled boxes if the order that the objects are placed in the boxes does not matter, and such that any box can have at most 4 books?

Example Template: How many ways can # {distinct OR identical} {books OR containers OR jars} be placed {in OR on} # {labeled OR unlabeled} {boxes OR shelves} if the order that the objects are placed {in the boxes OR on the shelf} {matters OR does not matter}, and such that any {box OR shelf} can have {any number of OR at most # OR at least #} {books OR containers OR jars}?

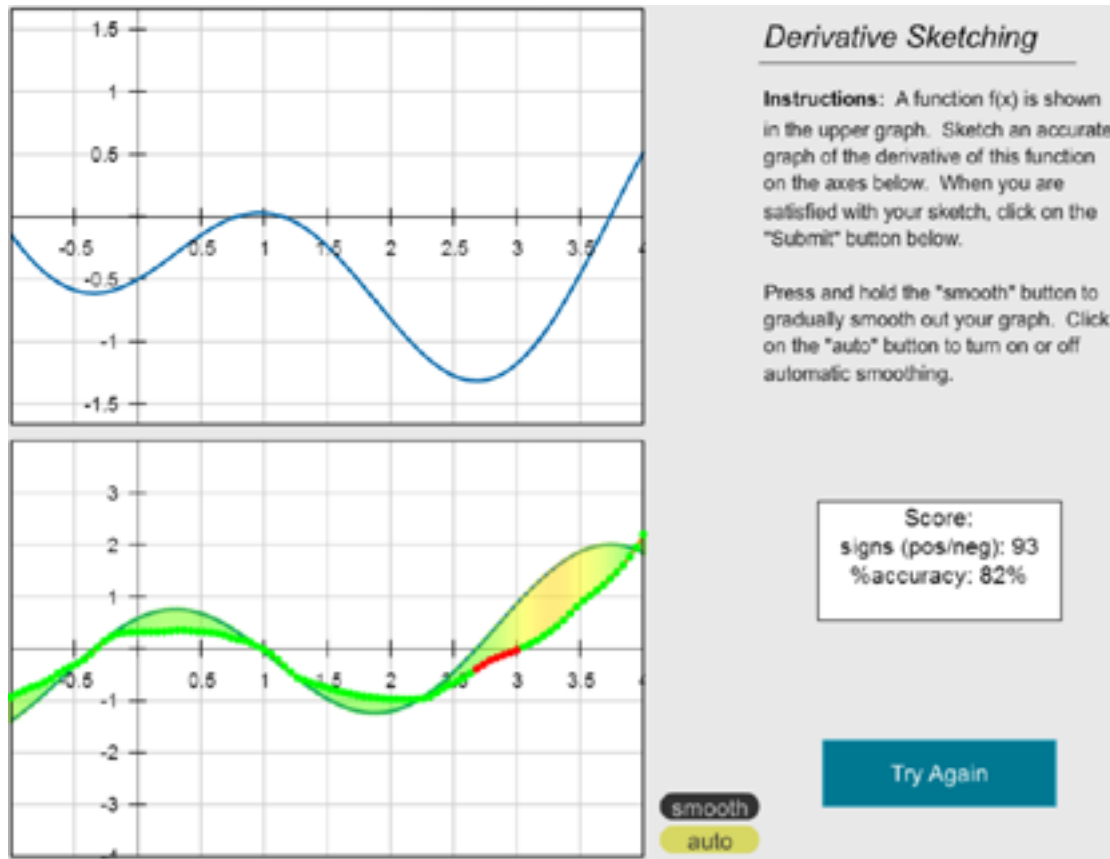


Figure 3: Example of a template generated graphical problem

There are several popular open-source interactive mathematics environments, such as IMathAS, WebWork and WIMS (Bokhove & Drijvers, 2010), that are template driven, but only in (Goehle, 2013) was any consideration given to gamification (and it was limited to the use of badges and levels).

In (Pollanen, 2007), we introduced the *Xero* interactive learning environment. It differs from other environments in that *Xero* was designed to create as highly randomized assignments as possible, so as to provide students with a considerable degree of challenge. An assignment of 30 questions is created by selecting 30 random templates in a random order from a much larger template pool. The parameters in each template were then randomly populated. Students were allowed to complete the assignment by answering questions in order. After each question was answered, a student would be informed by the software as to whether he or she got it correct or not, and would be shown a solution.

After completing the assignment, students were allowed to retake their assignment as many times as they wished. In many interactive learning environments, students are permitted to have multiple attempts at a question or to retake an entire assignment.

However, with *Xero*, for each assignment attempt a student would be presented with a completely new randomized assignment, which, with the high level of randomization in the design, would look quite different from that associated with the previous attempt.

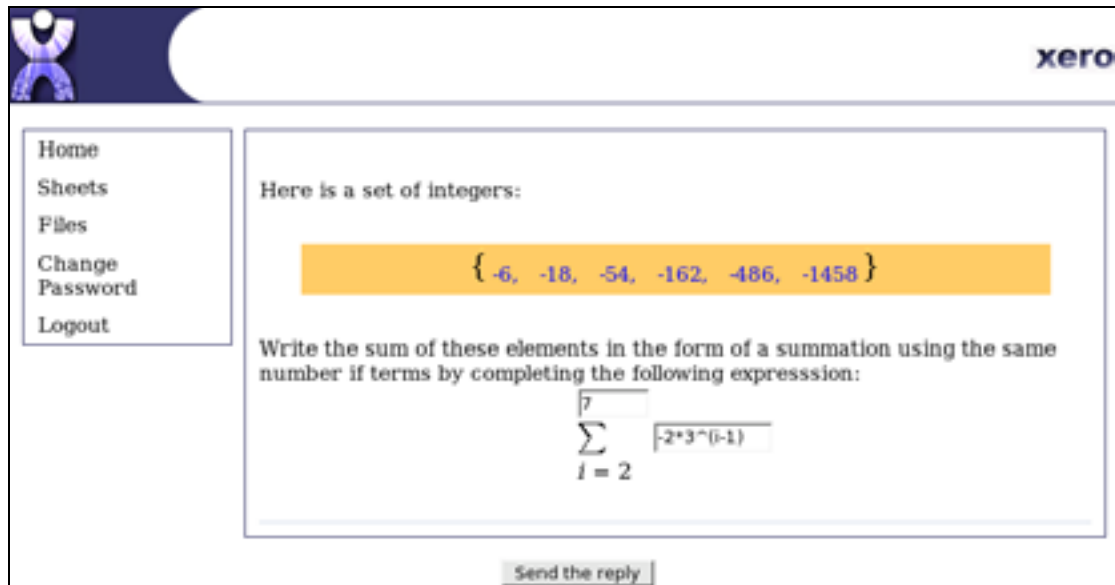


Figure 4: Screenshot of an exercise in the interactive homework environment *Xero*

The use of these online assignments in a service course was studied in (Pollanen, 2007), who compared them to traditional paper assignments. In surveys, it was revealed that students did 7-9 times as many questions on a *Xero* assignment as compared to a paper assignment. Students stated that the online assignments were quite difficult, yet they thought that they learned more from them, and students found them to be more enjoyable. The results in Table 1 show that students were very engaged by this format.

Interestingly, students commented that they found the assignments to be “fun and entertaining”, “like a challenging game”, and like a “game show” (Pollanen, 2007). Needless to say, this level of enthusiasm for mathematics in a service course is quite surprising. The reason for this appears to be that to improve the score a student would have to attempt a completely new challenging assignment. Each attempt takes a considerable amount of time. Only after a student had invested a lot of time did he or she know whether the score from the current attempt would beat the score from the previous attempt. Often the score on an attempt may be close to a best attempt, making for some nail-biting moments as the student is trying to beat his or her best score.

	<i>Assignment #1 (Paper)</i>	<i>Assignment #2 (Online)</i>	<i>About the Same</i>
<i>Average number of questions answered</i>	30	265	
<i>Which assignment was more difficult?</i>	13% (3/23)	74% (17/23)	13% (3/23)
<i>Which assignment was a better learning tool?</i>	17% (4/23)	74% (17/23)	9% (2/23)
<i>Which assignment did you enjoy more?</i>	14% (3/22)	82% (18/22)	5% (1/22)

Table 1: Summary of survey responses comparing interactive assignments in *Xero* to traditional paper-based assignment

The fact that students found the format of this assignment to be game-like is interesting in that the assignment consisted entirely of a series of mathematics questions, with no visible game-like elements. This means that it is efficient from both the developer and student perspectives. In the next section, we will explore how we can make this model even more engaging based on what has been learned from popular games.

5. Gamified Assignment Model

The initial version of *Xero* was intended to be a basic interactive and adaptive learning environment. It was not designed to be game-like, however it is perceived to be so by many students. The next version under development will attempt to extend its game-like characteristics.

Like the original version of *Xero*, it will use templates that can generate an unlimited number of questions. Each assignment will consist of a large pool of templates, with each instance of an assignment being a random subset of templates being populated with random parameters. Thus, two users opening an assignment would see very different, but equally difficult, assignments concerning the same topic. In surveys, many students commented on the high degree of randomization with statements like “I don’t think I EVER got the same question twice” (student’s emphasis).

Students are often accustomed to test question order approximating the order the material has been presented in class. It has been observed that scrambling the order of the material makes it much more difficult for students, and thus it requires that they learn it better.

Students writing an assignment are under different pressures than those writing a test. To take advantage of the testing effect, we propose to make the assignments more test-like. Like the original version of *Xero*, the questions must be answered in order, and once viewed a question must be answered in that session with the option for the instructor to impose a time-limit. The instructor may also specify the minimum number of questions that must be answered in a session, ranging from one to all.

Once a question has been answered, students will be presented with a solution. This utilizes the testing effect, which posits that a student is best able to correct misconceptions during the test itself. This is a form of assessment for learning, which has been shown to be very successful for learning mathematics (Hodgen & William, 2006). If a student gets the question wrong additional attempts can be made on a newer randomized version of the question created by that template. However, their maximum grade for that question for each subsequent attempt will be half that of the previous attempt. This is analogous to the Immediate Feedback Assessment Technique (Epstein, Epstein, & Brosvic, 2001), which uses multiple choice scratch cards while *Xero* allows freeform input of expressions. Showing the solution after each attempt was found to be popular among students, who made many comments of the following kind: “I was able to improve right away” and “I could figure out where I went wrong and not make the same mistake next time”.

Like the original version of *Xero*, students are provided with an unlimited number of attempts at the assignment. However, one weakness with the first version was that it did not include any concept of a catch-up. Thus, if a student were to realize that a subsequent attempt would not be better there could be little incentive to continue being engaged in the current attempt. To counter this possibility, we also include random bonus questions worth more marks.

Other options that could be included are lifelines. For example, a student might have a limited number of lifelines, such as an option to skip a question, or option to turn a free-form question into a multiple choice question.

6. Conclusion

Gamification has become a popular topic in recent years as online learning and methods for improving student engagement have come into focus. In this paper, we presented a model for gamification in post-secondary mathematics and statistics courses that relies on template driven software to be able to create an unlimited number of practice problems. We showed that it is feasible to create a game with few visible extrinsic game elements, so the student does not need to learn the rules of the game or spend time on gameplay. This model is reusable with different content and is thus efficient from both the perspectives of both instructors and students. Little work has, thus far, been done on gamifying mathematics learning environments, and so there is an opportunity to develop and further refine this model.

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