

MATHEMATICS 150

(2001-2002)

PROBLEM SET 5

Solutions may be submitted in class on **Monday, February 4** or may be submitted to the Instructor's office (CC F30) by 4:00 pm Tuesday, February 5 . *Print your name on the upper right-hand corner of the front page.*

1. A screening test for a particular type of treatable cancer has been criticised as being subject to diagnostic errors. This criticism is to be assessed on the basis of a probability model. It is assumed that there is a 2% chance that any member of the susceptible population selected at random would have this cancer. On the basis of some limited sampling, it has been assumed that there is only a 35% chance of obtaining a positive result with a person who has this cancer and there is a 1.25% chance of a (false) positive result with a person without the cancer.
 - a) If a member of the susceptible population is selected at random and tested, what is the probability that the person will produce a positive result?
 - b) If a member of the susceptible population is selected at random and tested and the test gives a positive result, what is the probability that the person has this cancer?
 - c) If a member of the susceptible population is selected at random and tested and the test gives a negative result, what is the probability that the person has this cancer?
2. An investor is considering making a single investment with one of two firms, *A* or *B*. In a simplified analysis, judging each firm's past experience and assumed abilities, the investor has generated a profile of potential profits on the investment with each firm and has assigned odds against each of the potential profits. The analysis produced the results tabulated below. Check the odds for consistency. **If the odds are inconsistent**, correct them using the method illustrated in the class example. Use the odds to develop corresponding probabilities and then revise these probabilities to the closest values such that each is of the form $n/20$ where n is an integer and such that the new probabilities are, themselves, consistent with probability rules. Using these revised probabilities, determine which firm would have the greatest expected percentage profit.

firm	potential profit (%)	odds against profit
A	15	10 to 1
	10	7 to 3
	0	1 to 1
	-10	9 to 1
B	30	17 to 3
	15	3 to 1
	10	3 to 2
	0	9 to 1
	-20	9 to 1

3.
 - a) If 17 of the 25 most recently investigated traffic accidents involved damage in excess of \$2500, and if the reports for 8 of these 25 accidents are to be selected as a sample, what is the probability that 6 of the 8 will involve damage in excess of \$2500?
 - b) If 680 of the 1000 most recently investigated traffic accidents involved damage in excess of \$2500, and if the reports for 8 of these 1000 accidents are to be selected as a sample, what is the probability that 6 of the 8 will involve damage in excess of \$2500?
 - c) If 3400 of the 5000 most recently investigated traffic accidents involved damage in excess of \$2500, and if the reports for 80 of these 5000 accidents are to be selected as a sample, what is the probability that over 60 of the 80 will involve damage in excess of \$2500?
4. If it is assumed that 15% of traffic accidents involve some form of injury,
 - a) what is the probability that over 4 of the next 20 accidents investigated will involve some form of injury?
 - b) what are the mean and variance of the possible numbers of accidents with some form of injury in the next 20 accidents investigated?
 - c) what is the probability that over 40 of the next 200 accidents investigated will involve some form of injury?

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5. If there is a probability $1/800$ that a randomly selected traffic accident report will be a report for an accident involving a fatality and if a random sample of 2000 traffic accident reports is to be selected,
 - a) how many of the reports would be “expected” to be reports for accidents involving a fatality?
 - b) what is the probability that at most 5 of the reports selected will be for accidents involving a fatality?
6. Vehicles arrive at a parking garage at random during shopping hours at a mean rate of three per minute. The ticket machine has become jammed and the lift bar at the entrance to the garage has become stuck in the closed position. It will take t minutes to release the machine jam so that the gate can work.
 - a) What is the probability that more than ten vehicles will arrive at the toll station before the gate is released if $t = 5$?
 - b) What is the probability that more than 50 vehicles will arrive at the station before the gate is released if $t = 15$?
7. In a shared computer system, individual users are allotted 40 MB of primary disk space for their data files, with overflow placed on secondary disks. It has been claimed that this allotment is insufficient and that a “substantial” proportion of users would exceed their disk quotas in any given month with the files that they would be expected to store with new forthcoming user requirements. Suppose that the monthly amounts of disk space needed to store the files generated for the new requirements would follow a normal distribution with a mean of 35 MB and a standard deviation of 5 MB.
 - a) What is the probability that, in a given month, an individual user would need disk space between 30 and 40 MB, inclusive?
 - b) What is the probability that an individual user’s monthly space needs would exceed the 40 MB primary disk space allotment?
 - c) In the ‘long run’ what percentage of users would need more than the allotted primary disk space?
 - d) If the allotment is to be changed so that only 5% of users would exceed their allotments, how much space should be allotted to each user?