MATHEMATICS 150

(2001-2002)

PROBLEM SET 4

Solutions are due on **Monday, December 3**. Solutions may be submitted in class or delivered **by 4:00 pm** to the instructor's office (CC F30). **Print your name on the upper right-hand corner of the front page.**

1. A prediction model is to be developed to predict daily coffee consumption in an office complex. The prediction model is to be based on the average number of people in the complex (a methodology has been developed to determine the average number), the number of hours of operation and the price of the coffee. Sample values from a number of complexes are listed below. The data are also available in a file **coffee.dat** in the usual way.

a) Use multiple regression to develop a linear prediction equation to predict coffee consumption from number of people, hours of operation and price. Predict consumption for a complex with 100 people that operates 10 hours per day with the price of coffee as $1.10.

b) It was decided that a multiplicative model should be used instead of a linear model. This is accomplished by converting all data to new values which are logarithms of original values. Use multiple regression to develop a linear prediction equation to predict the logarithm of the coffee consumption from the logarithms of the number of people, hours of operation and price. How much improvement in fit is there? Predict the logarithm of the consumption and, hence, the consumption for a complex with 100 people that operates 10 hours per day with the price of coffee as $1.10.

number of people	hours of operation	price	number of cups consumed
316	9	1.00	1094
361	9	1.25	1088
270	12	1.25	946
304	12	1.00	1285
268	10	1.00	956
289	10	1.00	1251
127	9	1.50	280
190	9	1.35	545
268	12	1.25	914
183	9	1.15	604
279	9	1.25	756
304	11	1.50	904
150	11	1.25	470
129	9	1.05	404
153	9	1.25	473
212	9	1.00	689
289	12	1.25	973
186	10	1.15	738
87	9	1.00	312
241	9	1.25	606
187	10	1.00	651
283	12	1.35	933
247	12	1.05	1016
170	9	1.25	377
356	10	1.00	1438
169	9	1.15	559
237	11	1.25	842
242	10	1.25	806
154	9	1.25	470
249	9	1.50	611
267	9	1.25	720
255	10	1.50	821
117	9	1.25	306
367	9	1.25	984

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2. Suppose that a record was kept of the number of reported cases of a disease over several years and the annual rates per 100,000 population were as follows:

year	1981	1982	1983	1984	1985	1986	1987	1988
rate	4.2	6.5	6.5	6.9	5.1	6.7	8.2	8.8
year	1989	1990	1991	1992	1993	1994	1995	
rate	6.9	8.7	8.8	12.0	15.4	18.4	19.6	

a) What was the percentage increase/decrease i) from 1983 to 1984? ii) from 1988 to 1989? iii) from the start to the end of the record?

b) Determine the actual annual average rate of increase from 1981 to 1995.

c) Using a transformed regression, determine the average rate of increase of the underlying trend and determine what would have been the predicted rates for 1996 and for 1997. (Note: it may be preferable to work with a time variable such as t = year – 1980.)

3. a) A software development firm has examined its recent experience to determine the number of hours of employee time that a 'typical' project requires for programming, for testing, for documentation preparation and for marketing strategy planning. Different professionals are involved in these stages of the project and are paid at different hourly rates. The numbers of hours required are tabulated below with the hourly rates (dollars) paid in the initial study year (year 0) and the first and second follow-up years (years 1 and 2.)

software development hours and hourly rates

		rate		
stage	hours	year 0	year 1	year 2
programming	1350	30.00	31.00	33.00
testing	270	18.00	19.00	21.00
documentation	180	24.00	24.00	25.00
marketing	64	20.00	23.00	27.00

The CPI for years 0, 1 and 2 for the company's area of operation were 138.1, 141.2 and 143.6

i) Determine a software development cost index for this firm for years 1 and 2 using year 0 as the base year.

ii) Using year 0 as the base year, what was the real cost of a 'typical' project for each of years 0, 1 and 2?

b) The firm also works on contracts for other organizations. The hours of staff time per quarter assigned to contract work has been estimated to follow a trend line $y = 960 + 32t$ where y is the quarterly number of hours of staff time and t is time *in quarters* from an initial time point five years earlier. The starting time is set at $t = 0$ and subsequent quarters produce increments of 1 so that quarter one of year 1 produces $t = 1$, quarter two of year 1 produces $t = 2$, quarter one of year 2 produces $t = 5$ etc . It is assumed that quarters 1, 2, 3, and 4 of any year have seasonal factors of 1.38, 0.96, 0.55, and 1.11, respectively.

i) Predict demand for each quarter of year 6. (Assume no cyclical effect.)

ii) If the centred moving average for quarter one of year 5 was 1512 and the actual number of hours was 2115, what was the individual empirical seasonal factor for that quarter?

iii) If the actual number of hours for quarter 3 of year 3 was 707, what was the seasonally adjusted value?

4. A major-equipment service facility has experienced seasonal variation according to the quarter of the year. A four-year record of the numbers of service calls is shown in the following table

	Year				
Quarter	1	2	3	4	5
I	272	288	336	392	384
II	144	152	184	200	213
III	88	88	120	128	122
IV	244	284	268	324	341

a) Sketch a graph of this time series.

b) Convert these values to seasonally adjusted values. In the determination of the seasonal factors, use *centred* moving averages and, unless you are using Minitab, use ordinary means (not modified means or medians) to average the individual empirical seasonal factors.

c) Plot the seasonally adjusted values on the graph in a).

d) Determine the forecasts for the four quarters of year 6.