

1. (Discovery Project on page 532) Arc length contest (60)

The curves shown are all examples of graphs of continuous functions f that have the following properties.

1. $f(0) = 0$ and $f(1) = 0$
2. $f(x) \geq 0$ for $0 \leq x \leq 1$
3. The area under the graph of f from 0 to 1 is equal to 1.

The lengths L of these curves, however, are different.

Try to discover formulas for two functions that satisfy the given conditions 1, 2, 3. Then calculate the arc length of each graph. The winning entry will be the one with the smallest length.

Note: The functions will have to be continuous too. You need to verify that the three conditions are satisfied by your functions. You will be marked on the optimality (the length of the shorter one of the two curves) and originality (how many copies of the same function are submitted) of your entries.

2. (Page 654, Exercise 44) When recording live performances, sound engineers often use a microphone with a cardioid pickup pattern because it suppresses noise from the audience. Suppose the microphone is placed 4 m from the front of the stage (as in the figure) and the boundary of the optimal pickup region is given by the cardioid $r = 8 + 8 \sin \theta$, where r is measured in meters and the microphone is at the pole. The musicians want to know the area they will have on stage within the optimal pickup range of the microphone. Answer their question. (40)