MATH 1100-A 2008 Quiz 9
Sections 4.2 to 4.4 Nov. 25, 2008

1. (3 pts) Find the intervals on which f is increasing or decreasing where

$$f(x) = \frac{x}{x^2+4}.$$

Solution:

$$f' = \frac{(x^2+4) - x(2x)}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2}$$
$$f' = 0 \Leftrightarrow x = 2 \text{ or } x = -2$$

Since

$$f'(-3) = \frac{4-(-3)^2}{\left((-3)^2+4\right)^2} < 0,$$

$f' < 0$ on the interval $(-\infty, -2)$. f is decreasing on $(-\infty, -2)$.

Since

$$f'(0) = \frac{4-0^2}{(0^2+4)^2} > 0,$$

$f' > 0$ on the interval $(-2, 2)$. f is increasing on $(-2, 2)$.

Since

$$f'(3) = \frac{4-3^2}{(3^2+4)^2} < 0,$$

$f' < 0$ on the interval $(2, \infty)$. f is decreasing on $(2, \infty)$. □

2. (2 pts) Find the limit

$$\lim_{x\to 0} \frac{x - \sin x}{x^2}.$$

Solution: Since $\lim_{x\to 0} x - \sin x = 0$ and $\lim_{x\to 0} x^2 = 0$, l'Hospital's Rule applies:

$$\lim_{x\to 0} \frac{x-\sin x}{x^2} = \lim_{x\to 0} \frac{1-\cos x}{2x}.$$

Since $\lim_{x\to 0} 1 - \cos x = 0$ and $\lim_{x\to 0} 2x = 0$, we apply l'Hospital's Rule again:

$$\lim_{x\to 0} \frac{1-\cos x}{2x} = \lim_{x\to 0} \frac{\sin x}{2} = 0.$$

□