MATH 1100-A 2008 Quiz 4

1. (2.5) Explain why $f(x)$ is discontinuous at $x = 2$ where

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & x < 2 \\ 2 & x = 2 \\ 3x - 2 & x > 2 \end{cases}.$$

Solution:

$$\begin{aligned} \lim_{x \to 2^-} f(x) &= \lim_{x \to 2^-} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2^-} \frac{(x-2)(x+2)}{x-2} \\ &= \lim_{x \to 2^-} (x+2) = 4. \end{aligned}$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (3x - 2) = 4.$$

Therefore,

$$\lim_{x \to 2} f(x) = 4 \neq f(2) = 2.$$

$f(x)$ is discontinuious at $x = 2$. $\square$

2. (2.5) Find the horizontal and vertical asymptotes (if they exist) of the curve

$$y = \frac{2x+1}{(x-1)(x+2)}.$$

Solution: Since

$$\begin{aligned} \lim_{x \to \infty} \frac{2x+1}{(x-1)(x+2)} &= \lim_{x \to \infty} \frac{2x+1}{x^2 + x - 2} \\ &= \lim_{x \to \infty} \frac{\frac{2}{x} + \frac{1}{x^2}}{1 + \frac{1}{x} - \frac{2}{x^2}} = 0, \end{aligned}$$

and

$$\lim_{x \to -\infty} \frac{2x+1}{(x-1)(x+2)} = 0$$

$y = 0$ is the horizontal asymptote.

$$\lim_{x \to 1^+} \frac{2x+1}{(x-1)(x+2)} = \infty$$

$$\lim_{x \to (-2)^-} \frac{2x+1}{(x-1)(x+2)} = -\infty$$

$x = 1$ and $x = -2$ are vertical asymptotes.

Note: One could consider the limits when $x \to 1^-$ and $x \to (-2)^+$ as well, but the above is sufficient for $x = 1$ and $x = -2$ to be the vertical asymptotes. However, the limit when $x \to 1$ (or $x \to -2$) is neither ∞ or $-\infty$ since it is not the same on both sides. $\square$