MATH 110 2008 Quiz 3
Sections 2.3, 2.4

1. (2.5) Evaluate the limit, if it exists.

$$\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}$$

Solution:

$$\begin{aligned}
&\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}\cdot\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}\\
=\ &\lim_{h\to 0}\frac{4+h-4}{h\left(\sqrt{4+h}+2\right)} == \lim_{h\to 0}\frac{h}{h\left(\sqrt{4+h}+2\right)}\\
=\ &\lim_{h\to 0}\frac{1}{\sqrt{4+h}+2}=\frac{1}{4}.
\end{aligned}$$

□

2. (2.5) Prove the statement using the ϵ, δ definition of limit.

$$\lim_{x\longrightarrow 2}(3x-2)=4.$$

Solution: 1. Choose δ.

For a given $\epsilon > 0$, we want whenever $|x-2|<\delta$,

$$\begin{aligned}
&|(3x-2)-4|<\epsilon\\
\Leftrightarrow\ &|3x-6|<\epsilon\\
\Leftrightarrow\ &3\,|x-2|<\epsilon\\
\Leftrightarrow\ &|x-2|<\frac{\epsilon}{3}
\end{aligned}$$

So we choose $\delta=\frac{\epsilon}{3}$.

2. Prove this δ works.

Whenever $|x-2|<\delta$,

$$\begin{aligned}
&|x-2|<\frac{\epsilon}{3}\\
\Leftrightarrow\ &3\,|x-2|<\epsilon\\
\Leftrightarrow\ &|3x-6|<\epsilon\\
\Leftrightarrow\ &|(3x-2)-4|<\epsilon
\end{aligned}$$

□