

MATH 110 2008 Quiz 20

March 24, 2009

Sections 10.2, 10.3.

1. Find the (x, y) -coordinates of the points on the curve $r = 3 \cos \theta$ where the tangent line is horizontal or vertical. (5 pts)

Solution:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\begin{aligned} \frac{dy}{d\theta} &= \frac{d}{d\theta} (r \sin \theta) \\ &= \frac{d}{d\theta} (3 \cos \theta \sin \theta) \\ &= -3 \sin^2 \theta + 3 \cos^2 \theta \\ &= 3 (\cos^2 \theta - \sin^2 \theta) \\ &= 3 \cos 2\theta. \end{aligned}$$

$$\begin{aligned} \frac{dx}{d\theta} &= \frac{d}{d\theta} (r \cos \theta) \\ &= \frac{d}{d\theta} (3 \cos^2 \theta) \\ &= 6 \cos \theta (-\sin \theta) \\ &= -3 \sin 2\theta. \end{aligned}$$

$$\begin{aligned} \frac{dy}{d\theta} &= 0 \Leftrightarrow 3 \cos 2\theta = 0 \\ \Leftrightarrow 2\theta &= \frac{\pi}{2}, -\frac{\pi}{2} \\ \Leftrightarrow \theta &= \frac{\pi}{4}, -\frac{\pi}{4}. \end{aligned}$$

$$\begin{aligned} \frac{dx}{d\theta} &= 0 \Leftrightarrow -3 \sin 2\theta = 0 \\ \Leftrightarrow 2\theta &= 0, \pi \\ \theta &= 0, \frac{\pi}{2}. \end{aligned}$$

The points where there is a horizontal tangent line have $\theta = \frac{\pi}{4}, -\frac{\pi}{4}$. Therefore either

$$\begin{aligned} x &= r \cos \left(\frac{\pi}{4} \right) \\ &= 3 \cos^2 \left(\frac{\pi}{4} \right) = \frac{3}{2}, \\ y &= r \sin \left(\frac{\pi}{4} \right) \\ &= 3 \cos \left(\frac{\pi}{4} \right) \sin \left(\frac{\pi}{4} \right) \\ &= \frac{3}{2}; \end{aligned}$$

or

$$\begin{aligned}
 x &= r \cos \left(-\frac{\pi}{4} \right) \\
 &= 3 \cos^2 \left(-\frac{\pi}{4} \right) = \frac{3}{2} \\
 y &= r \sin \left(-\frac{\pi}{4} \right) \\
 &= 3 \cos \left(-\frac{\pi}{4} \right) \sin \left(-\frac{\pi}{4} \right) \\
 &= -\frac{3}{2}.
 \end{aligned}$$

At the points $\left(\frac{3}{2}, \frac{3}{2}\right)$ and $\left(\frac{3}{2}, -\frac{3}{2}\right)$ there are horizontal tangent lines.

Similarly, the points where there is a vertical tangent line have $\theta = 0, \frac{\pi}{2}$. Either

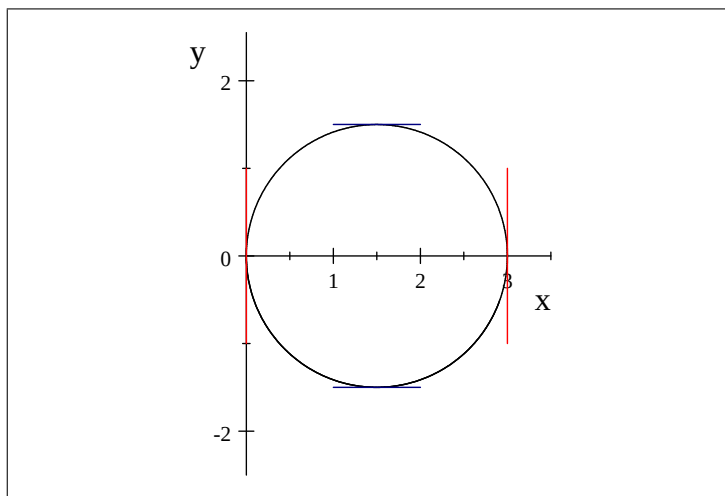
$$\begin{aligned}
 x &= r \cos 0 \\
 &= 3 \cos^2 0 = 3 \\
 y &= r \sin 0 = 0
 \end{aligned}$$

or

$$\begin{aligned}
 x &= r \cos \frac{\pi}{2} = 0 \\
 y &= r \sin \frac{\pi}{2} \\
 &= 3 \cos \frac{\pi}{2} \sin \frac{\pi}{2} = 0.
 \end{aligned}$$

At the points $(3, 0)$ and $(0, 0)$ there are vertical tangent lines.

$$(x - 1.5)^2 + y^2 = 1.5^2$$



□