MATH 1100-A 2008 Quiz 19 March. 17, 2009 Sections 8.2, 10.1.

1. (2 pts) Set up, but do not evaluate, an integral for the area of the surface obtained by rotating the curve $y = x^3$, $1 \le x \le 4$, about the x-axis.

Solution: The area is

$$S = \int_{1}^{4} 2\pi x^{3} \sqrt{1 + (3x^{3})^{2}} dx$$

2. (3 pts) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ where

$$\begin{cases} x = t^3 + 2t \\ y = t^2 - 1 \end{cases}$$

Solution:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{3t^2 + 2}.$$

$$\frac{d^2y}{dx^2} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}} = \frac{\frac{2(3t^2+2)-2t(6t)}{(2t^2+2)^2}}{3t^2+2}$$
$$= \frac{6t^2+4-12t^2}{(2t^2+2)^3}$$
$$= \frac{4-6t^2}{(2t^2+2)^3}.$$