

MATH 1100-A 2008 Quiz 18  
March. 10, 2009  
Sections 8.1.

(5 pts) Find the length of the curve

$$y = \sqrt{4 - x^2}, \quad 0 \leq x \leq 2.$$

*Solution:*

$$\begin{aligned} L &= \int_a^b \sqrt{1 + (y')^2} dx \\ &= \int_0^2 \sqrt{1 + \left(\frac{-2x}{2\sqrt{4-x^2}}\right)^2} dx \\ &= \int_0^2 \sqrt{1 + \frac{x^2}{4-x^2}} dx \\ &= \int_0^2 \sqrt{\frac{4}{4-x^2}} dx = \int_0^2 \frac{2}{\sqrt{4-x^2}} dx \end{aligned}$$

Let  $x = 2 \sin \theta$ ,  $dx = 2 \cos \theta d\theta$ ,  $\sqrt{4 - x^2} = \sqrt{4 - 4 \sin^2 \theta} = 2 \cos \theta$ .

$$\begin{aligned} &= \int_0^{\frac{\pi}{2}} \frac{2}{2 \cos \theta} 2 \cos \theta d\theta \\ &= 2 \int_0^{\frac{\pi}{2}} d\theta = 2\theta \Big|_0^{\frac{\pi}{2}} = \pi. \end{aligned}$$