(5 pts) Find the length of the curve

$$y = \sqrt{4 - x^2}, \ 0 \le x \le 2.$$

Solution:

$$
\begin{aligned}
L &= \int_a^b \sqrt{1 + (y')^2} dx \\
&= \int_0^2 \sqrt{1 + \left(\frac{-2x}{2\sqrt{4 - x^2}}\right)^2} dx \\
&= \int_0^2 \sqrt{1 + \frac{x^2}{4 - x^2}} dx \\
&= \int_0^2 \sqrt{\frac{4}{4 - x^2}} dx = \int_0^2 \frac{2}{\sqrt{4 - x^2}} dx
\end{aligned}
$$

Let $x = 2\sin\theta$, $dx = 2\cos\theta d\theta$, $\sqrt{4 - x^2} = \sqrt{4 - 4\sin^2\theta} = 2\cos\theta$.

$$
\begin{aligned}
&= \int_0^{\frac{\pi}{2}} \frac{2}{2\cos\theta} 2\cos\theta d\theta \\
&= 2\int_0^{\frac{\pi}{2}} d\theta = 2\theta|_0^{\frac{\pi}{2}} = \pi.
\end{aligned}
$$