MATH 1100-A 2008 Quiz 16
Feb. 24, 2009
Sections 7.3, 7.4.

Evaluate

1. (2.5 pts)

$$\int \frac{t^3}{\sqrt{25-t^2}}dt$$

Solution: (Method 1): Let $t = 5\sin\theta$, $dt = 5\cos\theta d\theta$.

$$\begin{aligned}\int \frac{(5\sin\theta)^3}{\sqrt{25-(5\sin\theta)^2}}5\cos\theta d\theta &= 5^4\int \frac{\sin^3\theta\cos\theta}{5\cos\theta}d\theta \\ &= 5^3\int \sin^3\theta d\theta \\ &= 5^3\int \left(1-\cos^2\theta\right)\sin\theta d\theta\end{aligned}$$

(Let $u = \cos\theta$, $du = -\sin\theta d\theta$)

$$\begin{aligned}&= -5^3\int \left(1-u^2\right)du = -5^3\left(u-\frac{u^3}{3}\right)+C \\ &= 5^3\left(\frac{\cos^3\theta}{3}-\cos\theta\right)+C \\ &= 5^3\left(\frac{(25-t^2)^{\frac{3}{2}}}{3\cdot 5^3}-\frac{\sqrt{25-t^2}}{5}\right)+C\end{aligned}$$

where $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{t}{5}\right)^2} = \frac{\sqrt{25-t^2}}{5}$. □

(Method 2): Let $u = 25-t^2$. $du = -2tdt$. $t^2 = 25-u$.

$$\begin{aligned}\int \frac{t^3}{\sqrt{25-t^2}}dt &= \int \frac{t^2\cdot t}{\sqrt{25-t^2}}dt \\ &= -\frac{1}{2}\int \frac{25-u}{\sqrt{u}}du \\ &= -\frac{1}{2}\int \frac{25}{\sqrt{u}}du+\frac{1}{2}\int \frac{u}{\sqrt{u}}du \\ &= -25\sqrt{u}+\frac{1}{3}u^{\frac{3}{2}}+C \\ &= -25\sqrt{25-t^2}+\frac{1}{3}\left(25-t^2\right)^{\frac{3}{2}}+C.\end{aligned}$$

□

2. (2.5 pts)

$$\int \frac{3x+5}{(x+4)(x-3)}dx$$

Solution: We let

$$\frac{3x+5}{(x+4)(x-3)} = \frac{A}{x+4} + \frac{B}{x-3}.$$

$$3x+5 = A(x-3) + B(x+4)$$

Method 1: Let $x = 3$, we have

$$14 = B \cdot 7 \Longrightarrow B = 2.$$

Let $x = -4$, we have

$$-7 = A(-7) \Longrightarrow A = 1.$$

Method 2:

$$\begin{aligned} 3x+5 &= A(x-3) + B(x+4) \\ &= (A+B)x - 3A + 4B \end{aligned}$$

We have

$$\begin{aligned} A+B &= 3 \\ -3A+4B &= 5 \end{aligned}$$

Solving this system of equations, we get

$$A = 1, B = 2.$$

Therefore,

$$\begin{aligned} \int \frac{3x+5}{x^2+x-12}dx &= \int \left(\frac{1}{x+4} + \frac{2}{x-3}\right)dx \\ &= \ln(x+4) + 2\ln|x-3| + C. \end{aligned}$$

□