MATH 1100-A 2008 Quiz 10
Sections 4.5 and 4.7 Dec. 2, 2008

1. Let $f(x) = x^3 - 9x^2$.Find the following (if they exist):

   (a) if f is even or odd;

   (b) the intervals of increasing and decreasing and local maximums and local minimums;

   (c) concavity and points of inflection.

   No need to graph the function.

   Solution: (a)
$$f(-x) = -x^3 - 9x^2$$
   $f(-x) \neq f(x)$. $f(-x) \neq -f(x)$. f is neither even or odd.

   (b)
$$f' = 3x^2 - 18x = 3x(x-6)$$
   $f' = 0 \Leftrightarrow x = 0$ or $x = 6$. We consider the intervals $(-\infty, 0), (0, 6)$ and $(6, \infty)$.

   $f'(-1) = 3(-1)(-1-6) = 21 > 0$. f' is positive on $(-\infty, 0)$.

   $f'(1) = -15 < 0$. f' is negative on $(0, 6)$.

   $f'(7) = 21 > 0$. f' is positive on $(6, \infty)$.

   $f(x)$ is increasing on $(-\infty, 0)$ and $(6, \infty)$ and decreasing on $(0, 6)$. $f(x)$ has a local maximum at $x = 0$ and local minimum at $x = 6$.

   (c) $f'' = 6x - 18 = 6(x-3)$. f'' is negative on $(-\infty, 3)$ and positive on $(3, \infty)$. f is concave downward on $(-\infty, 3)$ and concave upward on $(3, \infty)$. f has a point of inflection at $x = 3$. $\square$

2. Find two positive numbers whose product is 200 and whose sum is a minimum.

   Solution: Let the two numbers be x and y and the sum is
$$S = x + y$$
   Since
$$\begin{aligned} xy &= 200, \\ y &= \frac{200}{x} \end{aligned}$$
   and
$$S = x + \frac{200}{x}.$$
$$S' = 1 - \frac{200}{x^2}$$

Let $S' = 0$. We have

$$\begin{aligned} 1 - \frac{200}{x^2} &= 0 \\ \frac{x^2 - 200}{x^2} &= 0 \\ x^2 - 200 &= 0 \\ x &= \sqrt{200}. \end{aligned}$$

The two numbers are $x = \sqrt{200}$ and $y = \frac{200}{x} = \sqrt{200}$. $\square$