Calculus 110B Review Problems

WARNING!!

The following 7 X 17 = 119 problems cover most of the subjects we talked about in class but not all (see your instructor!).

Do all parts!! Even though some types of questions are repeated from part to part, NEW problems appear along the way! For example, Part 1 does not include a length of curve question!!

Good review problems are also those from older tests and exams (section A and B)! See website!!

GOOD LUCK!!

Part 1

1. Use the Intermediate Value Theorem to demonstrate that the equation
$$x = \frac{1}{4}e^{(1-x^2)}$$
has a solution on $-1 \leq x \leq 1$.

2. Find the value of the limit:
$$\lim_{p \to 0} p^2 \ln(p)$$

3. A “v” shaped trough with depth 1 m, width (across the top of the ”v”) 2 m and length 7 m is filled from a tap through which water flows at 6 cm^3/s. What is the rate of increase of water depth in the trough just before it overflows?

4. Find y' if $x^4 + y^4 = 7xy$.

5. A rectangular box with a square base and open top must have a volume of 64 cm^3. Find the dimensions of the box that minimizes the amount of material needed to manufacture it.

6. If $f'(x) = \frac{x+1}{x-1}$ find the intervals where $f(x)$ is concave up and the intervals where it is concave down.

7. Show that
$$1 - x \leq \cos x \leq 1 + x, \qquad \text{for every } x \geq 0$$
by applying the Mean Value Theorem to some function f over the interval $[0, x]$.

8. Find the function f such that:
$$f^{'}(x) = 3x^{-2}, \quad f(1) = -f(-1).$$

9. Evaluate the following indefinite integral using the substitution rule:
$$\int x(4 + x^2)^{10} dx$$

10. Evaluate the following indefinite integral using partial fractions:

$$\int \frac{x}{x^2 - 3x + 2}\, dx$$

11. Evaluate the following indefinite integral using the method of integration by parts:

$$\int t^3 \ln t dt$$

12. A particle moves along a line so that its acceleration function at time t is $a(t) = 2t$ (in m/s^2). Let $v(t)$ denote the velocity function of the particle at time t (in m/s). Knowing that $v(0) = -1$, find the displacement of the particle during the time period $0 \leq t \leq 2$ and the total distance traveled during the same time period.

13. Sketch the graph of

$$f(x) = x + \sqrt{1 - x^2}$$

(domain, intercepts, asymptotes, monotony (increase/decrease intervals), local max/min values, concavity and points of inflection)

14. Find the area between the following parabolae: $y^2 = 6x$ and $x^2 = 6y$.

15. Solve the ODE (initial-value problem):

$$x\frac{dy}{dx} - \frac{y}{x+1} = x, \; y(1) = 0, \; x > 0$$

16. Show that the sequence defined by $a_1 = 1$,

$$a_{n+1} = 3 - \frac{1}{a_n},$$

for all $n \geq 1$ is increasing and $a_n < 3$ for all $n \geq 1$. Deduce that the $(a_n)_{n\geq 1}$ is convergent and find its limit.

17. Find the radius of convergence and interval of convergence of the series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt{n}}$$

Part 2

1. Use the Intermediate Value Theorem to show that if $f(x) = x^3 - 5x$ then $f(x) = 6$ for some x.

2. Consider a function $x(t)$ which satisfies the formula

$$[x(t)]^2 - 6t = t^2 x(t)$$

and goes through the point $(t, x) = (2, -2)$. Calculate dx/dt at the point $(2, -2)$.

3. Compute:

$$\lim_{x \to \infty} \frac{x^2 - 1}{e^x}$$

4. A man standing on a train moving at 100 km/h measures his distance from another man standing 3 km away from the track. How fast is this distance increasing when their separation is 4 km?

5. Find the coordinates of the point on the parabola $2x + y^2 = 0$ that is closest to the point $(0, -3/2)$.

6. Show that the equation

$$x^7 + x + 5000 = 0$$

has exactly one solution.

7. Let $h'(t) = e^{1/t^2}$. Find all the intervals for which the graph of $h(t)$ is concave down.

8. Find the function f if

$$f''(x) = \cos(x), \quad f'(0) = 1, \quad f(0) = -1$$

9. Compute the following definite integral using the substitution rule:

$$\int_0^{\pi^2/4} \frac{\sin\sqrt{x}}{\sqrt{x}} dx$$

10. Evaluate the following indefinite integral using partial fractions:

$$\int \frac{x^2}{x^2 - 5x + 6} dx$$

11. Evaluate the following indefinite integral using the method of integration by parts:

$$\int \ln^2 x dx$$

12. A particle moves along a line so that its acceleration function at time t is $a(t) = 2t$ (in m/s^2). Let $v(t)$ denote the velocity function of the particle at time t (in m/s). Knowing that $v(0) = -4$, find the displacement of the particle during the time period $0 \leq t \leq 5$ and the total distance traveled during the same time period.

13. Sketch the graph of

$$f(x) = x - \ln(1 + x)$$

(domain, intercepts, asymptotes, monotony (increase/decrease intervals), local max/min values, concavity and points of inflection)

14. Find the volume of the solid obtained by rotating the region bounded by the given curve about the x-axis:

$$y = \sec x,\ y = 1,\ x = -1,\ x = 1$$

15. Solve the ODE (initial-value problem):

$$\frac{dx}{dt} = \frac{2t + 1}{2(x - 1)},\ x(0) = -1$$

16. Compute:

$$\lim_{n\to\infty} \frac{\sin n}{\sqrt{n}}$$

if it exists.

17. Evaluate the indefinite integral as a power series:

$$\int \tan^{-1}(x^2) dx$$

Part 3

1. Use the Intermediate Value Theorem to demonstrate that the equation
$$x = \frac{1}{4}e^{(1-x^2)}$$
has a solution on $-1 \leq x \leq 1$.

2. Consider $g(x) = x^{2/5}$. Evaluate $g'(x)$ for $x \neq 0$. Describe the behaviour of the tangent line as x approaches 0.

3. Calculate the slope of the tangent line to the graph of the relation $x^2y - e^{-x}\ln y = 0$ at the point $(x, y) = (-1, e)$.

4. A balloon is filled with helium from a pump at a constant rate of 2 m^3/s. When the balloon has a volume of 2 m^3 what is the speed at which the surface of the balloon is moving outwards? Hint: The volume of a sphere of radius R is $V = \frac{4}{3}\pi R^3$.

5. A poster is to have an area of 200 m^2 with 1 m margins at the bottom and sides and a 2 m margin at the top. What dimensions will give the maximum printed area?

6. Show that the equation
$$x^{2001} + 2000x + 1999 = 0$$
has exactly one solution.

7. Where is the graph of $f(x)$ concave up on $[1, 1 + 2\pi]$ if
$$f'(x) = \frac{1}{\cos(x-1)} \text{ ?}$$

8. Evaluate the indefinite integral
$$\int (\cos x - 2\sin x)dx$$

9. Evaluate the indefinite integral using the substitution rule:
$$\int e^{\sin\theta}\cos\theta d\theta$$

10. Evaluate the following indefinite integral using partial fractions:

$$\int \frac{s+3}{s^3-s}\,ds$$

11. Evaluate the following indefinite integral using the method of integration by parts:

$$\int u^3 e^u du$$

12. A particle moves along a line so that its acceleration function at time t is $a(t) = 2t+1$ (in m/s^2). Let $v(t)$ denote the velocity function of the particle at time t (in m/s). Knowing that $v(0) = -2$, find the displacement of the particle during the time period $0 \leq t \leq 3$ and the total distance traveled during the same time period.

13. Sketch the graph of

$$f(x) = x^2 \ln x$$

(domain, intercepts, asymptotes, monotony (increase/decrease intervals), local max/min values, concavity and points of inflection)

14. Find the length of the curve:

$$y = \ln(\cos x),\ 0 \leq x \leq \frac{\pi}{4}$$

15. Study the improper integral (compute if convergent):

$$\int_0^1 \frac{\ln x}{\sqrt{x}} dx$$

16. Compute:

$$\lim_{n\to\infty} (3^n + 7^n)^{\frac{1}{n}}$$

if it exists.

17. Use the binomial series to find the Maclaurin series of

$$f(x) = \frac{1}{\sqrt{1+x^3}}.$$

Use it to evaluate $f^{(9)}(0)$.

Part 4

1. Consider the equation $x^2 = \sqrt{x+1}$. Use the Intermediate Value Theorem to prove that there is a root of this equation on $(1, 2)$.

2. Find the equation of the tangent line of the graph

$$p(t) = e^{t-2} + \cos(\pi t)$$

   at the point $(2, 2)$.

3. Two people run opposite directions along a circular track of 50 m radius at the constant speed of $\pi/5\ m/s$. At what speed are they moving apart when they are a quarter of the track apart?

4. Find y' if $y^3 x = \cos(2x + y)$.

5. Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the the x-axis and lying on the parabola $y = 4 - x^2$.

6. Apply the Mean Value Theorem to the function $f(x) = \cos x$ to prove the inequality:

$$|\cos a - \cos b\,| \leq |a - b|$$

   for all real numbers a and b.

7. Find the function f if

$$f^{'}(x) = 1 + \frac{1}{x^2}, \quad x > 0, f(1) = 1$$

8. The equation $e^{1/x} = x$ has an unique solution in the interval $(1, 3)$. If $x_1 = 1$, find the second approximation, x_2, of this solution, using Newton's method.

9. Evaluate the indefinite integral using the substitution rule:

$$\int x^3(1 - x^4)^5 dx$$

10. Evaluate the following indefinite integral using partial fractions:
$$\int \frac{1}{x^3 - 2x^2 + x}\,dx$$

11. Evaluate the following indefinite integral using the method of integration by parts:
$$\int e^{-u}\sin(2u)du$$

12. A particle moves along a line so that its acceleration function at time t is $a(t) = 2t - 1$ (in m/s^2). Let $v(t)$ denote the velocity function of the particle at time t (in m/s). Knowing that $v(0) = 2$, find the displacement of the particle during the time period $0 \le t \le 3$ and the total distance traveled during the same time period.

13. Sketch the graph of
$$f(x) = e^{-\frac{1}{x^2}}$$
if $x \neq 0$ and $f(0) = 0$.
(domain, intercepts, asymptotes, monotony (increase/decrease intervals), local max/min values, concavity and points of inflection)

14. The given curve is rotated about the y-axis. Find the area of the resulting surface:
$$x = e^{2y},\ 0 \le y \le 1/2$$

15. Study the convergence of the improper integral using the Comparison Theorem:
$$\int_1^\infty \frac{\sin^2 x}{x^2}dx$$

16. Determine whether the series is convergent or divergent. If it is convergent, find its sum:
$$\sum_{n=0}^{\infty}\left(\frac{1}{2^n} + \left(\frac{3}{\pi}\right)^n\right)$$

17. Find the Maclaurin series for
$$f(x) = x^3 e^{-x}$$

Part 5

1. Use the Intermediate Value Theorem to demonstrate that
$$\ln x = \sin\left(\frac{\pi x^2}{2}\right)$$
has a root on $1 \leq x \leq \sqrt{3}$.

2. If $\sqrt{p} + \sqrt{q} = 3$ find dp/dq at the point $(p, q) = (4, 1)$.

3. A 15 ft ladder propped up against the wall begins to slide. When the top of the ladder is 12 ft from the ground, it is moving down at a rate of 3 ft/sec. What is the speed of the other end of the ladder along the ground?

4. Compute:
$$\lim_{x\to\infty} (e^x + x^2)^{1/x}$$

5. A rectangular storage container with an open top is to have 4000 m^3. The length of its base is twice its width. Material for the base costs \$15 per square meter. Material for the sides costs \$5 per square meter. Find the cost for the materials for the cheapest such container.

6. Show that the equation
$$x^{1999} + 2000x + 2001 = 0$$
has exactly one solution.

7. Given that $p'(z) = (e^{z^2} + e^{-z^2})/2$, find all the values of z where the graph of $p(z)$ is concave up.

8. Find the function f such that:
$$f''(x) = 2 + x^3 + x^6, \quad f'(0) = f(0) = 0$$

9. Evaluate the following indefinite integral using the substitution rule:
$$\int xe^{-x^2}\,dx$$

10. Evaluate the following indefinite integral using partial fractions:

$$\int \frac{u^2+4}{u^3+2u}\,du$$

11. Evaluate the following definite integral using the method of integration by parts:

$$\int_0^1 (t^2+1)e^{-t}dt$$

12. A particle moves along a line so that its acceleration function at time t is $a(t) = t+3$ (in m/s^2). Let $v(t)$ denote the velocity function of the particle at time t (in m/s). Knowing that $v(0) = 5$, find the displacement of the particle during the time period $0 \leq t \leq 20$ and the total distance traveled during the same time period.

13. Sketch the graph of

$$f(x) = \sin x - \tan x$$

(domain, intercepts, asymptotes, monotony (increase/decrease intervals), local max/min values, concavity and points of inflection)

14. Find the length of the curve:

$$x = 3t - t^3,\ y = 3t^2,\ 0 \leq t \leq 2.$$

15. Study the convergence of the improper integral using the Comparison Theorem:

$$\int_1^\infty \frac{dx}{x+e^{3x}}$$

16. Determine whether the series is convergent or divergent:

$$\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2+1}$$

17. Find the sum of the following series:

$$\sum_{n=2}^{\infty} n(n-1)t^n,\ |t| < 1$$

Part 6

1. Show using the Intermediate Value Theorem that the equation

$$x^3 \ln x = e^x$$

possesses a root in the interval $(1, 3)$.

2. Find the equation of the tangent line of the graph

$$y = 3x^2 + \ln(2x + 1)$$

at $x = 3$.

3. A kite 200 ft above the ground moves horizontally at a speed of 10 ft/s. At what rate is the angle between the string and the horizontal decreasing when 400 ft of string have been let out.

4. Find y' if $x \cos y + y \sin x = c$, where c is a constant real number.

5. Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius r. (The results should be in terms of r.)

6. Show that the equation

$$4x - 100 + \sin(\pi x/2) = 0$$

has exactly one solution.

7. If $f'(x) = \frac{1}{(\ln x)^2}$, find all the intervals for which the graph of $f(x)$ is concave downward.

8. Evaluate the integral

$$\int_0^2 (x^2 - |x - 1|)dx$$

9. Evaluate the indefinite integral using the substitution rule:

$$\int \frac{x}{\sqrt{x + 1}}\, dx$$

10. Evaluate the following indefinite integral using partial fractions:

$$\int \frac{1}{t^2(t-1)^2}\, dt$$

11. Evaluate the following definite integral using the method of integration by parts:

$$\int_1^4 \ln \sqrt{u}\, du$$

12. A particle moves along a line so that its acceleration function at time t is $a(t) = 2t + 3$ (in m/s^2). Let $v(t)$ denote the velocity function of the particle at time t(in m/s). Knowing that $v(0) = -4$, find the displacement of the particle during the time period $0 \le t \le 4$ and the total distance traveled during the same time period.

13. Sketch the graph of

$$f(x) = x^{5/3} - 5x^{2/3}$$

(domain, intercepts, asymptotes, monotony (increase/decrease intervals), local max/min values, concavity and points of inflection)

14. Find the length of the curve:

$$x = e^t - t,\ y = 4e^{t/2},\ 0 \le t \le 1.$$

15. Solve the ODE:

$$1 + xy = xy'$$

16. Determine whether the series is convergent or divergent:

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 2n + 2}$$

17. Use series to evaluate

$$\lim_{x\to\infty} x^2(1 - e^{-1/x^2})$$

Part 7

1. Consider $\sqrt{x+y} + \sqrt{x-y} = 4$. Find the tangent line at the point $(x, y) = (5, 4)$.

2. Compute:
$$\lim_{x \to 0} \frac{x}{\tan^{-1}(2x)}$$

3. Two sides of a triangle have lengths 13 m and 15 m. The angle between them is increasing at a rate of 0.06 rad/s. Find the rate at which the third side is increasing when the angle between the sides of fixed length is $\pi/3$.

4. If $f(x) = x^3 - x^2 + x + 3$, show that there is a number c such that $f(c) = 13$.

5. Given a right triangle with hypotenuse of length 10 m, find the lengths of the other two sides for which the area of the triangle is maximal.

6. Does there exist a differentiable function f such that $f(1) = -3$, $f(4) = 0$ and $f'(x) > 1$ for all x? Justify your answer using the Mean Value Theorem.

7. Evaluate the indefinite integral
$$\int x^3(1 - x^4)dx$$

8. The equation $e^{1/x} = x^2$ has a unique solution in the interval $(1, 2)$. If $x_1 = 1$, find the second approximation, x_2, of this solution using Newton's method.

9. Evaluate the definite integral using the substitution rule:
$$\int_0^{\sqrt{\pi}} x \cos(x^2)\, dx$$

10. Evaluate the following indefinite integral using partial fractions:
$$\int \frac{u^3}{u^3 + 1}\, du$$

11. Evaluate the following definite integral using the method of integration by parts:

$$\int_0^1 s7^s ds$$

12. A particle moves along a line so that its acceleration function at time t is $a(t) = 2t + 3$ (in m/s^2). Let $v(t)$ denote the velocity function of the particle at time t (in m/s). Knowing that $v(1) = -14$, find the displacement of the particle during the time period $1 \leq t \leq 4$ and the total distance traveled during the same time period.

13. Sketch the graph of

$$f(x) = \ln(\tan^2 x)$$

(domain, intercepts, asymptotes, monotony (increase/decrease intervals), local max/min values, concavity and points of inflection)

14. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$:

$$x = t + \cos t,\ y = \sin(2t)$$

15. Solve the ODE:

$$xy' + xy + y = e^{-x},\ x > 0.$$

16. Find the values of p for which the series is convergent:

$$\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^p}$$

17. Find the Taylor series of

$$f(x) = 3 + \cos x$$

at $x = \pi/3$.