MATH 155H, Fall 2003, Assignment 4
11 problems X 10 points each = 110 points total

1. The cdf of a continuous random variable X is given by

$$F_X(x) = \alpha + \beta tan^{-1}\left(\frac{x}{2}\right), \quad -\infty < x < \infty.$$

Determine the constants α and β and the pdf of X.

2. Let X be a continuous random variable with pdf

$$f_X(x) = \frac{e^{-|x|}}{2}, \quad -\infty < x < \infty.$$

Find $P(-2 < X < 1)$.

3. Let X be a continuous random variable with pdf

$$f_X(x) = \begin{cases} \frac{4x^3}{15} & \text{if } 1 \leq x \leq 2 \\ \\ 0 & \text{otherwise} \end{cases}$$

Find the pdfs of $Y = e^X$, $Z = X^2$, and $W = (X-1)^2$.

4. Let X be a continuous random variable with pdf

$$f_X(x) = \begin{cases} \frac{2}{x^3} & \text{if } 1 \leq x \\ \\ 0 & \text{otherwise} \end{cases}$$

Find $E[X]$ and $Var[X]$.

5. Suppose that b is a random number from the interval $(-3, 3)$. What is the probability that the quadratic equation $x^2 + bx + 1 = 0$ has at least one real root?

6. Let $X \sim N(\mu, \sigma^2)$. Calculate the pdf of $Y = e^X$.

7. The amount of soft drink in a bottle is a normal random variable. Suppose that in 7% of the bottles containing this soft drink there are less than 15.5 ounces, and in 10% of them there are more than 16.3% ounces. What are the mean and standard deviation of the amount of soft drink in a randomly selected bottle?

8. Guests arrive at a hotel, in accordance with a Poisson process, at a rate of 5 per hour. Suppose that for the last 10 minutes no guest has arrived. What is the probability that:

(a) the next one will arrive in less than 2 minutes;

(b) from the arrival of the tenth to the arrival of the eleventh guest takes more than 2 minutes?

9. Customers arrive at a restaurant at a Poisson rate of 12 per hour. If the restaurant makes profit only after 30 customers have arrived, what is the expected length of time until the restaurant starts to make profit?

10. For a random variable X the mgf is

$$M_X(t) = \frac{1}{81}(e^t + 2)^4.$$

Find $P(X \le 2)$ and $E(X^3)$. What is the distribution of X?

11. A psycologist wants to estimate μ, the mean IQ of the students of a university. To do so, she takes a sample of size n of the students and measures their IQ's. Then she finds the average of these numbers. If she believes that the IQ's of these students are independent random variables with variance 170, how large a sample should she choose to be 98% sure that her average is accurate within ± 0.2?