MATH 155H Fall 2003 Assignment #1

1. If $P(A) = 1/3$, $P(B) = 1/4$ and $P(A \cap B) = 1/6$, find the following probabilities:

   (a) $P(\bar{A})$
   (b) $P(\bar{A} \cup B)$
   (c) $P(A \cup \bar{B})$
   (d) $P(\bar{A} \cup \bar{B})$

2. A fair coin is tossed 4 times.

   (a) Show a sample space for the experiment, showing each possible sequence of tosses.

   (b) Suppose the sample points are equally likely and a running count is made of the number of heads and the number of tails tossed. What is the probability that the heads count always exceeds the tails count?

3. If $P(A) = .9$ and $P(B) = .8$, show that $P(A \cap B) \geq .7$.

4. At an art exhibition there are 12 paintings of which 10 are original. A visitor selects a painting at random and before he decides to buy, he asks the opinion of an expert about the authenticity of the painting. The expert is right in 9 out of 10 cases on average.

   (a) Given that the expert decides that the painting is authentic, what is the probability that this is really the case? (*Hint:* Use Bayes' Theorem.)

   (b) If the expert decides that the painting is a copy, then the visitor returns it and chooses another one; what is the probability that his second choice is an original? (*Hint:* This probability depends on whether or not the expert was right in the first case.)

5. An urn contains 3 white, 3 black and 4 red balls.

   (a) A ball is drawn at random. Find the probability that the ball is (i) red, (ii) black or white, (iii) not white.

   (b) Two balls are drawn at random without replacement. Find the probability that (i) both balls are red, (ii) one is white and one is black.

   (c) Two balls are drawn at random one at a time with replacement. Find the probability that (i) both are red, (ii) one is white and one is black.

6. A bin contains 20 items, 6 of which are defective. Three items are selected at random. Find the probability that there is at least one defective among the three selected.

7. A pizza shop offers eight toppings. If no topping is used more than once, in how many ways can a three-topping pizza be formed? If the choice of three toppings is done at random, what is the probability that one of the toppings is pepperoni?

8. A common computer programming rule is that names of variables must be between 1 and 8 characters long. The first character can be any one of the 26 letters, while successive characters can also be any of the 10 digits. For example, allowable variable names are A, $B4$ and $M3477K$. How many different variable names are possible?

9. A small pond contains 50 fish, 10 of which have been tagged. If a random catch of 7 fish is made, what is the probability that the catch contains exactly two tagged fish?

A curious note: This simple idea is actually used in order to estimate the number of fish of a certain species in a body of water. Suppose you did not know that there were 50 fish, and you denote by N the unknown number of fish. Then replacing 50 by N in the formula you just used gives a function of N which can then be maximized to find the value of N which makes what you observed most probable. That number would be your best "estimate" for N.