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THE MATHEMATICS OF PRINCIPAL-AGENT PROBLEM WITH ADVERSE SELECTION

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Abstract. This paper studies the existence and characterization of optimal solutions to the principal-agent problem with adverse selection for both discrete and continuous problems. The existence results are derived by the abstract concepts of differentiability and convexity. It is known that under the Spence Mirrlees condition, the principal-agent problem can be reduced to a simpler problem which can be solved explicitly. But not much results on the solution are known when the Spence Mirrlees condition does not hold. For the problem without the Spence Mirrlees condition, we give some sufficient conditions to verify the linear independence and the Mangasarian Fromovitz constraint qualification.

1. **Introduction.** The principal-agent problem is a problem which frequently occurs in economics (contract theory) and also political science [2, 7, 11]. It arises when a principal (e.g., firm, organization, employer, seller) assigns a task to an agent (e.g., worker, employee, buyer) through a contract. And the goal of the principal is to assign the contract in a way that maximizes his profit while compensating the agent for performing the task required from him. This problem has been discussed extensively in many mathematics and economics literature, e.g., [6, 12]. There are two types of principal-agent problems based on information asymmetry, that is, when one party of the contract has more or better information than the other. These are the moral-hazard or hidden action (i.e., the case where the agent can take an action unobservable to the principal), and the adverse selection or hidden knowledge (i.e., the case where some relevant information of the agent is unobservable to the principal).

In this paper, we focus on the principal-agent problem with adverse selection. It is based on an economic contract that relates a principal and an agent such that some relevant characteristics (defined here by θ) of the agent is unobservable by the principal [6]. Here, without loss of generality, we assume that the principal is the owner of a restaurant and his customers are the agents. The asymmetric information is the customer's taste which is not known to the owner. In this case, the only available information to the owner of the restaurant are the proportions of

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customers with specific taste-type. In our model, θ represents the taste of customers which belongs to some bounded domain $\Theta \subset \mathbb{R}^p$. The customer with taste θ goes to the restaurant and orders a food with quality $q \in \mathbb{R}^m_+$ and pays a monetary transfer $t \in \mathbb{R}_+$ for the price of the food. Let $h(\theta, q)$ denote the satisfaction of the customer of type θ buying the food with quality q. Then the welfare or utility of this agent (defined here by "a") is

$$U_a(\theta) = h(\theta, q(\theta)) - t(\theta).$$

Roughly speaking, $U_a(\theta)$ quantifies how much a customer with taste θ enjoys the food with quality q, knowing that he spends the amount t for it. If $C(q)$ represents the cost of producing the food with quality q, then the utility of the principal (defined here by "p") is

$$U_p(\theta) = t(\theta) - C(q(\theta)).$$

Here $U_p(\theta)$ can be viewed as the profit that the owner of the restaurant makes in selling the food with quality q to the customer with taste θ. Since the goal of the owner is to make more profit, then he tries to anticipate the customers' choices so that each customer reveals his taste by choosing the food that is targeted for him. Therefore, the principal's utility $U_p(\theta)$ is subject to some constraints, called incentive compatible constraints, meaning that the customers are given incentive to reveal their real tastes. Mathematically, the incentive compatible constraints can be represented as

$$h(\theta, q(\theta)) - t(\theta) \geq h(\theta, q(\theta')) - t(\theta') \qquad \forall \theta, \theta' \in \Theta.$$

So, the principal-agent problem can be formulated as follows:

$$\begin{aligned} (PA) \quad \max_{q(\theta), t(\theta)} \quad & \int_\Theta (t(\theta) - C(q(\theta)))f(\theta)d\theta && && (1) \\ \text{s.t.} \quad & h(\theta, q(\theta)) - t(\theta) \geq 0, && \forall \theta \in \Theta && \text{(IR)} \\ & h(\theta, q(\theta)) - t(\theta) \geq h(\theta, q(\theta')) - t(\theta'), && \forall \theta, \theta' \in \Theta && \text{(IC)} \end{aligned}$$

where $f(\theta) \in L^\infty(\Theta)$ is the probability density function representing the distribution of customers' tastes. The first set of constraints represents individual rationality constraint (IR for short) meaning that customers go to the restaurant only if they receive at least zero level of utility, otherwise the customers will choose to go to another restaurant. The second set of constraints is the incentive compatibility constraints (IC for short) described above.

In the discrete case, when there are n customers, by using the argument discussed earlier, the problem can be formulated as:

$$\begin{aligned} (PA)_d \quad \max_{q_i, t_i} \quad & \sum_{i=1}^n (t_i - C(q_i))f_i && && (2) \\ \text{s.t.} \quad & h(\theta_i, q_i) - t_i \geq 0, && \forall i = 1, \dots, n && \text{(IR)} \\ & h(\theta_i, q_i) - t_i \geq h(\theta_i, q_j) - t_j && \forall i, j = 1, \dots, n && \text{(IC).} \end{aligned}$$

Existence of solutions to problems (1) and (2), as well as characterization of solutions have been one of the main issues discussed in the past thirty years by many economists and mathematicians [4, 12]. The concept of adverse selection in contract theory was first introduced and analyzed by Mussa and Rosen [9] in a model of nonlinear monopoly pricing. Maskin and Riley [8] addressed this problem from a different perspective, by using a graphical method. Carlier [4] studied the general existence of solutions to the principal-agent problem when the principal is an employer and the agent is his employee. For the characterization of the solution to this problem, it is customary to require that the utility of the agent satisfies some technical condition, namely the *Spence Mirrlees condition* [2, 10]. This condition means that the marginal rate of substitution between quality and money is either increasing or decreasing with respect to the customer's taste. In fact the Spence Mirrlees condition allows the principal-agent problem to be reduced to a simpler problem which can be solved explicitly [2].

Although there are many utility functions that do not satisfy the Spence Mirrlees condition, very little is known about the solution to the principal-agent problem in these cases. One notable exception is the paper by Araujo and Moreira [1] where a special class of continuum of type problem is solved explicitly under a relaxed condition called U-shaped condition.

One of the goals of this paper is to study the principal-agent problem in the discrete case without imposing the Spence Mirrlees condition. More precisely, this paper has two main objectives. The primal one is to obtain existence results to problems (1) and (2) by adapting the proof used by Carlier [4] for the model of employers and employees. The second one is to study the problem (2) without imposing the Spence Mirrlees condition.

The paper is organized as follows. In section 2, we introduce some sufficient conditions that guarantee the existence of the solution to the principal-agent problem in continuum of type problem. This is followed by the discrete problem in section 3. In section 4, we study the discrete problem when the Spence Mirrlees condition does not hold.

2. **Existence of Solutions.** In this section, we study existence of solutions to the principal-agent problem (1). We will adapt the proof of Carlier [4] to our model of a monopolist and his customers. Hereafter, we assume that the taste, θ, of a customer belongs to some open and bounded convex subset Θ of $\mathbb{R}^p$ with a C^1 boundary. We denote by $\bar{\Theta}$ the closure of Θ, and by $h(\theta, q) : \bar{\Theta} \times \mathbb{R}^m_+ \to \mathbb{R}$, the satisfaction of the customer of type θ ordering the food with quality q. The customer's utility is then given by $U_a(\theta, q, t) = h(\theta, q) - t$, and that of the owner is $U_p(t, q) = t - C(q)$, where $t \in \mathbb{R}_+$ denotes the price of the food with quality q, and $C(q)$ is the cost of producing this food. We assume that $\|f\|_{L^\infty(\Theta)} > 0$.

In [4], Carlier presented a general existence result to the principal-agent problem in the context of employers and employees. Here, we propose to adapt the hypotheses in his proof to our model which deals with a monopolist (i.e, the owner of a restaurant) and his consumers (i.e., the customers). Because of the similarities between these two models, most of the propositions in [4] will be applicable here. Before stating the main theorem, we first introduce some definitions and propositions which will be useful in the proof of the existence theorem. For their proofs, we refer to [4]. Below, we will rewrite the principal's problem as an optimal control problem.

2.1. **The principal's problem as an optimal control problem.** Here and below, we assume that (q, t) is a contract, meaning that it is a pair of functions $(q, t) : \Theta \to \mathbb{R}^m_+ \times \mathbb{R}_+$.

Definition 1. (Implementability)
A function $q : \Theta \to \mathbb{R}^m_+$ is called implementable if there exists a function $t : \Theta \to \mathbb{R}_+$

such that the pair (q,t) is an incentive compatible contract (IC), i.e.,

$$h(\theta, q(\theta)) - t(\theta) \geq h(\theta, q(\theta')) - t(\theta'), \qquad \forall (\theta, \theta') \in \Theta^2.$$

Definition 2. The potential associated with a contract (q,t) is the function $U_{q,t} : \Theta \to \mathbb{R}$ defined by

$$U_{q,t}(\theta) = h(\theta, q(\theta)) - t(\theta).$$

Definition 3. (h-convexity)
A function $V : \Theta \to \mathbb{R} \cup \{+\infty\}$ is h-convex if there exists a nonempty subset A of $\mathbb{R}^m_+ \times \mathbb{R}_+$ such that

$$V(\theta) = \sup_{(q,t)\in A} \{h(\theta, q) - t\}.$$

The notion of h-convexity is a generalization of the concept of convexity. To illustrate, suppose A is a convex set and $h(\theta, q) = \theta q$. Then, the above definition gives us the classic definition of convexity as follows:

$$\begin{aligned}
V(\lambda\theta_1 + (1-\lambda)\theta_2) &= \sup_{(q,t)\in A} \{h(\lambda\theta_1 + (1-\lambda)\theta_2, q) - t\} \\
&= \sup_{(q,t)\in A} \{\lambda\theta_1 q + (1-\lambda)\theta_2 q - \lambda t - (1-\lambda)t\} \\
&= \sup_{(q,t)\in A} \{\lambda(\theta_1 q - t) + (1-\lambda)(\theta_2 q - t)\} \\
&\leq \sup_{(q,t)\in A} \{\lambda(\theta_1 q - t)\} + \sup_{(q,t)\in A} \{(1-\lambda)(\theta_2 q - t)\} \\
&\leq \lambda V(\theta_1) + (1-\lambda) V(\theta_2).
\end{aligned}$$

Definition 4. (h-differentiability)
Let $V : \Theta \to \mathbb{R} \cup \{+\infty\}$. A vector $q \in \mathbb{R}^m_+$ is called a h-subgradient of V at θ if

$$V(\theta') \geq V(\theta) + h(\theta', q) - h(\theta, q), \qquad \forall \theta' \in \Theta.$$

The set of all h-subgradients of V at $\theta \in \Theta$ is called the h-subdifferential of V at θ and is denoted by $\partial^h V(\theta)$. If $\partial^h V(\theta) \neq \emptyset$ for $\theta \in \Theta$, we say that V is h-subdifferentiable at θ.

It would give us the classic definition of subgradient if we suppose that V is a convex function, Θ is an open convex set and $d^T = \frac{h(\theta', q) - h(\theta, q)}{\theta' - \theta}$.
The following proposition gives the relation between implementability and the notions of h-convexity and h-subdifferentiability.

Proposition 1. *A function $q : \Theta \to \mathbb{R}^m_+$ is implementable if and only if there exists some h-convex and h-subdifferentiable mapping $V : \Theta \to \mathbb{R}$ such that $q(\theta) \in \partial^h V(\theta)$ for all $\theta \in \Theta$* [4].

From now on, we assume the following hypotheses:

H1. $h \in C^0(\bar{\Theta} \times \mathbb{R}^m_+, \mathbb{R})$ and for every $q \in \mathbb{R}^m_+$, $\theta \mapsto h(\theta, q)$ is nondecreasing i.e. for all α and β in Θ^2, if $\alpha_i \geq \beta_i$ for all $i = 1, ..., p$ (shortly $\alpha \geq \beta$) then $h(\alpha, q) \geq h(\beta, q)$.
H2. For every $(\theta, q) \in \Theta \times \mathbb{R}^m_+$, $\frac{\partial h}{\partial \theta}(\theta, q)$ exists, and the map $\frac{\partial h}{\partial \theta}(\cdot, \cdot)$ is continuous with respect to both arguments. Moreover, for every compact subset K of $\Theta \times \mathbb{R}^m_+$, there exists $k > 0$ such that for all $((\theta, q), (\theta', q)) \in K^2$

$$\| \frac{\partial h}{\partial \theta}(\theta, q) - \frac{\partial h}{\partial \theta}(\theta', q) \| \leq k \| \theta - \theta' \| .$$

H3. For every $M > 0$, there exists $r > 0$ such that for all $(\theta, q) \in \Theta \times \mathbb{R}_+^m$

$$\|q\| \geq r \Rightarrow \sum_{i=1}^{n} \frac{\partial h}{\partial \theta_i}(\theta, q) \geq M.$$

Remark 1. From (H1), we have that

$$V(\theta) := \sup_{(q,t)\in A} \{h(\theta, q) - t\}$$

is also nondecreasing in θ.

The following proposition gives the relation between h-convexity and h-differentiability.

Proposition 2. *Let $V : \Theta \to (-\infty, +\infty]$ be h-convex. If $K \subset \Theta$ is compact, $\delta > 0$ and $R > 0$ satisfy $K + \delta\bar{B}(0,1) \subset \Theta$, and $|V(\theta)| \leq R$ for all $\theta \in K + \delta\bar{B}(0,1)$, (where $\bar{B}(0,1)$ is the closed unit ball of R^p), then*

1. *V is h-subdifferentiable at every point of K.*
2. *There exists a positive constant $M(R, K, \delta)$ such that for all $\theta \in K$ and $q \in \partial^h V(\theta)$, we have $\|q\| \leq M(R, K, \delta)$* [4].

Remark 2. As a result of the proposition 2, if V is h-convex and locally bounded, then the set valued map $\partial^h V(\cdot)$ takes non-empty compact values.

Definition 5. A function V is called locally semi-convex if and only if for all convex compact subsets K of Θ there exists $\lambda > 0$ such that $V_\lambda(\cdot) := V(\cdot) + \lambda\|\cdot\|^2$ is convex in K. Any $\lambda > 0$ for which this property holds is called a semi-convexity modulus of V in K.

The next proposition will be used in the proof of the existence theorem. It states that h-convex potentials are locally semi-convex.

Proposition 3. *Assume that V and K are as in proposition 2. If furthermore K is convex, then V is locally semi-convex in K. In particular, any locally bounded h-convex mapping V is locally semi-convex in Θ* [4].

The following proposition relates h-subdifferentiability to the classical notion of gradient.

Proposition 4. *Let $V : \Theta \to \mathbb{R}$. Assume $q \in \partial^h V(\theta)$, where $\theta \in \Theta$ and V is differentiable at θ. Then, $\nabla V(\theta) = \frac{\partial h}{\partial \theta}(\theta, q)$* [4].

Remark 3. Combining Propositions 2, 3 and 4 and Rademacher's Theorem [3], we obtain that every locally bounded h-convex potential V is almost everywhere differentiable, everywhere h-subdifferentiable so that

$$\nabla V(\theta) = \frac{\partial h}{\partial \theta}(\theta, q), \text{ for almost every } \theta \in \Theta, \text{ for every } q \in \partial^h V(\theta).$$

Now, we can rewrite the principal-agent problem (1) as a variational problem with a h-convexity constraint:

$$(PA) \begin{cases} \inf & \Pi(q,t) := \int_\Theta [C(q) - t(\theta)] f(\theta) d\theta \\ s.t. & (q,t) \text{ incentive-compatible} \\ & h(\theta, q(\theta)) - t(\theta) \geq 0, \quad \forall \theta \in \Theta \end{cases}.$$

Proposition 5. *The principal-agent problem (1) is equivalent to the following problem:*

$$(PA')\begin{cases} \inf & J(q,V) := \int_\Theta [C(q) - h(\theta, q(\theta)) + V(\theta)] f(\theta) d\theta \\ s.t. & V \textit{ is h-convex,} \\ & q(\theta) \in \partial^h V(\theta), \quad \forall \theta \in \Theta \\ & V(\theta) \geq 0, \quad \forall \theta \in \Theta \end{cases}$$

Proof. The proof follows directly from Proposition 1. □

2.2. **Compactness.** In this section, by $V : \Theta \to \mathbb{R}$ h-convex, we mean that there exists a h-convex function $\tilde{V} : \Theta \to \mathbb{R}$ such that $\tilde{V} = V$ almost everywhere in Θ.
$\omega \subset\subset \Theta$ means that the closure of ω is included in Θ.
The proof of the next proposition can be found in Carlier [5].

Proposition 6. *Let (u_n) be a sequence of convex functions in Θ such that for every open convex set ω with $\omega \subset\subset \Theta$, the following holds:*

$$\sup_n \|u_n\|_{W^{1,1}(\omega)} < +\infty.$$

Then, there exists a function $\bar{u}$ that is convex in Θ, a measurable subset A of Θ and a (non relabelled) subsequence of (u_n), such that:

1. *(u_n) converges to $\bar{u}$ uniformly on compact subsets of Θ,*
2. *(∇u_n) converges to $\nabla \bar{u}$ pointwise on A and $\dim_H(\Theta \setminus A) \leq n - 1$, where $\dim_H(\Theta \setminus A)$ is the Hausdorff dimension of $\Theta \setminus A$. In particular, (∇u_n) converges to $\nabla \bar{u}$ almost everywhere in Θ.*

This proposition extends to h-convex functions as follows:

Proposition 7. *Let (V_n) be a sequence of h-convex functions in Θ such that the following holds:*

$$\sup_n \|V_n\|_{W^{1,1}(\Theta)} < +\infty.$$

Then, there exists a function $\overline{V} \in W^{1,1}(\Theta)$, that is h-convex in Θ, a measurable subset A of Θ and a (non relabelled) subsequence of (V_n), such that:

1. *(V_n) converges to $\overline{V}$ uniformly on compact subsets of Θ,*

2. *(∇V_n) converges to $\nabla \overline{V}$ pointwise in A and $\dim_H(\Theta \setminus A) \leq n-1$. In particular, (∇V_n) converges to $\nabla \overline{V}$ almost everywhere in Θ.*

Proof is available in [4].

2.3. **Existence result for a linear cost function.** First, we show the existence result for linear cost functions, then we will extend the argument to more general cost functions. In this section, we assume that $C(q) = \langle p, q \rangle$, where $p \in \mathbb{R}^m_+$ denotes the range of food prices. Then, the principal-agent problem becomes:

$$(PA)\begin{cases} \inf & \Pi(q,t) := \int_\Theta [\langle p, q \rangle - t(\theta)] f(\theta) d\theta \\ s.t. & (q,t) \text{ is incentive-compatible} \\ & h(\theta, q(\theta)) - t(\theta) \geq 0, \forall \theta \in \Theta, \end{cases}$$

or equivalently

$$(PA')\begin{cases} \inf & J(q,V) := \int_\Theta [\langle p, q \rangle - h(\theta, q(\theta)) + V(\theta)] f(\theta) d\theta \\ s.t. & V \text{ is h-convex,} \\ & q(\theta) \in \partial^h V(\theta), \forall \theta \in \Theta \\ & V(\theta) \geq 0, \forall \theta \in \Theta. \end{cases}$$

In addition to the previously mentioned hypotheses, we assume the following technical hypotheses:

H4. There exist $\alpha \leq 1$, $a > 0$ and $b \in \mathbb{R}$, such that for all $(\theta, q) \in \Theta \times \mathbb{R}_+^m$, we have

$$h(\theta, q) \leq a\|q\|^\alpha - b.$$

and if $\alpha = 1$, $a < \min_{1 \leq i \leq m} p_i$.

H5. There exist $\beta \in (0, \alpha), c > 0, d \in \mathbb{R}$ such that for all $(\theta, q) \in \Theta \times \mathbb{R}_+^m$

$$\|\frac{\partial h}{\partial \theta}(\theta, q)\| \leq c\|q\|^\beta + d.$$

Under the above assumptions, the following existence result holds.

Theorem 1. *(PA') admits at least one solution.*

Proof. Consider an arbitrary V that is both h-convex and locally bounded. Then, Proposition 2 and the measurable selection theorem imply that both set-valued maps $\partial^h V(\cdot)$ and $\Phi_V : \theta \to \underset{\partial^h V(\theta)}{\operatorname{argmin}}\{-h(\theta, \cdot) + \langle p, \cdot\rangle\}$ are non-empty, compact-valued and admit measurable selections.

Let (V_n, q_n) be a minimizing sequence of (PA'). Without loss of generality, we may assume that for all n, q_n is measurable and $q_n(\theta) \in \Phi_{V_n}(\theta)\ \forall\theta \in \Theta$. Then,

$$|\int_\Theta (V_n(\theta) - h(\theta, q_n(\theta)) + \langle p, q_n(\theta)\rangle) f(\theta) d\theta| \leq C, \tag{2.1}$$

where C is a positive constant.

Since $V_n(\theta)$ and $f(\theta)$ are both positive, we have

$$\int_\Theta (-h(\theta, q_n(\theta)) + \langle p, q_n(\theta)\rangle) f(\theta) d\theta \leq C.$$

By adding $\langle p, q_n(\theta)\rangle$ to both sides of the inequality in Hypothesis (H4), we have

$$\int_\Theta (-a\|q_n(\theta)\|^\alpha + \langle p, q_n(\theta)\rangle) f(\theta) d\theta \leq \int_\Theta (-h(\theta, q_n(\theta)) - b + \langle p, q_n(\theta)\rangle) f(\theta) d\theta \leq C',$$

where $C' = C - b$, $C' > 0$ by choosing C large enough.

Equivalently,

$$-a\int_\Theta \|q_n(\theta)\|^\alpha f(\theta) d\theta + \int_\Theta \langle p, q_n(\theta)\rangle f(\theta) d\theta \leq C'. \tag{2.2}$$

Let $M = \min_{1 \leq i \leq m} p_i > 0$. Then

$$\begin{aligned} \int_\Theta \langle p, q_n(\theta)\rangle f(\theta) d\theta &= \int_\Theta \sum_{i=1}^m p_i q_n^i f(\theta) d\theta \\ &\geq M \sum_{i=1}^m \int_\Theta q_n^i(\theta) f(\theta) d\theta \\ &= M \int_\Theta \|q_n(\theta)\| f(\theta) d\theta. \end{aligned} \tag{2.3}$$

Inserting (2.3) into (2.2) yields

$$-a\int_\Theta \|q_n(\theta)\|^\alpha f(\theta) d\theta + M\int_\Theta \|q_n(\theta)\| f(\theta) d\theta \leq C'. \tag{2.4}$$

At this step we consider two different cases: $\alpha < 1$ and $\alpha = 1$. First assume $\alpha < 1$. Then by Young's inequality we have

$$\begin{aligned}\|q_n(\theta)\|^\alpha &= \delta\|q_n(\theta)\|^\alpha \frac{1}{\delta}\\ &\leq \frac{(\delta\|q_n(\theta)\|^\alpha)^\eta}{\eta} + \frac{|\frac{1}{\delta}|^{\eta'}}{\eta'}\end{aligned}$$

where η and η' are positive real numbers such that $\frac{1}{\eta} + \frac{1}{\eta'} = 1$. Now, by choosing $\eta = \frac{1}{\alpha}$, we have

$$\|q_n(\theta)\|^\alpha \leq \alpha\delta^{\frac{1}{\alpha}}\|q_n(\theta)\| + \frac{1-\alpha}{\delta^{\frac{1}{1-\alpha}}}. \tag{2.5}$$

Inserting (2.5) into (2.4) yields

$$(M - a\alpha\delta^{\frac{1}{\alpha}})\int_\Theta \|q_n(\theta)\|f(\theta)d\theta - \frac{a(1-\alpha)}{\delta^{\frac{1}{1-\alpha}}}\int_\Theta f(\theta)d\theta \leq C'.$$

Consequently,

$$(M - a\alpha\delta^{\frac{1}{\alpha}})\int_\Theta \|q_n(\theta)\|f(\theta) \leq C'',$$

where $C'' = C' + \frac{a(1-\alpha)}{\delta^{\frac{1}{1-\alpha}}}$.
Choosing δ small enough so that $M - a\alpha\delta^{\frac{1}{\alpha}} > 0$ and using that $\|f\|_{L^\infty(\Theta)} > 0$, we have that (q_n) is bounded in $L^1(\Theta, \mathbb{R}^m_+)$.

Now we consider the case that $\alpha = 1$. Rewriting equation (2.4) for $\alpha = 1$ gives

$$(M - a)\int_\Theta \|q_n(\theta)\|f(\theta)d\theta \leq C'.$$

We know from (H4) that $M - a > 0$. Dividing both sides of the above equation by $(M - a)$ and using the fact that $\|f\|_{L^\infty(\Theta)} > 0$ implies that (q_n) is bounded is $L^1(\Theta, \mathbb{R}^m_+)$. Moreover, by (H4) we know that

$$h(\theta, q_n(\theta)) \leq a\|q_n(\theta)\|^\alpha - b. \tag{2.6}$$

At the same time using Young's inequality we have

$$\|q_n(\theta)\|^\alpha \leq \frac{(\|q_n(\theta)\|^\alpha)^\eta}{\eta} + \frac{1}{\eta'}$$

where η and η' are positive real numbers such that $\frac{1}{\eta} + \frac{1}{\eta'} = 1$. Inserting the above equation into (2.6) implies

$$h(\theta, q_n(\theta)) \leq a\left(\frac{\|q_n(\theta)\|^{\alpha\eta}}{\eta} + \frac{1}{\eta'}\right) - b.$$

Choosing $\eta = \frac{1}{\alpha}$ and using the fact that (q_n) is bounded in L^1, then $h(\theta, q_n)$ is bounded in L^1. Using this fact and that (q_n) is bounded in $L^1(\Theta)$ and $\|f\|_{L^\infty(\Theta)} > 0$, then (2.1) ensures that (V_n) is also bounded in $L^1(\Theta, \mathbb{R}_+)$. For all n, V_n is locally bounded. From Propositions 3 and 4, we deduce that for all n and for almost every $\theta \in \Theta$,

$$\nabla V_n(\theta) = \frac{\partial h}{\partial \theta}(\theta, q_n(\theta)).$$

Using Hypothesis (H4) we get,

$$\begin{aligned}\|\nabla V_n\| &= \|\frac{\partial h}{\partial \theta}(\theta, q_n(\theta))\| \\ &\leq c\|q\|^\beta + d, \quad \beta \in (0, \alpha) \\ &\leq c(1 + \|q_n(\theta)\|) + d \quad \text{ a.e. in } \Theta.\end{aligned}$$

Thus, ∇V_n is bounded in $L^1(\Theta)$ and V_n satisfies the assumptions of Proposition 6 Consequently, we may now assume that

$$\begin{cases} (V_n) \text{ converges in } L^1(\Theta) \text{ and uniformly on compact subsets of } \Theta, \\ (\nabla V_n) \text{ converges a.e. to } \nabla \bar{V}, \end{cases}$$

where $\bar{V} \in W^{1,1}(\Theta, \mathbb{R}_+)$ is h-convex.
Finally, define $\bar{q}(.)$ as a measurable selection of $\Phi_{\bar{V}}(\cdot)$.
First, since (V_n) converges to $\bar{V}$ in $L^1(\Theta)$ and $f \in L^\infty(\Theta)$ we have

$$\lim_n \int_\Theta V_n(\theta) f(\theta) d\theta = \int_\Theta \bar{V}(\theta) f(\theta) d\theta. \tag{2.7}$$

Fatou's Lemma yields,

$$\liminf_n \int_\Theta [-h(\theta, q_n(\theta)) + C(q_n)] f(\theta) d\theta \geq \int_\Theta \liminf_n [-h(\theta, q_n(\theta)) + C(q_n(\theta))] f(\theta) d\theta. \tag{2.8}$$

Let us define for all fixed θ,

$$\alpha(\theta) := \liminf_n \{-h(\theta, q_n(\theta)) + C(q_n(\theta))\}.$$

Since $(q_n(\theta))$ is bounded in L^1 by Hypothesis (H4), up to a subsequence, we may assume that

$$\begin{cases} \alpha(\theta) = \liminf_n -h(\theta, q_n(\theta)) + C(q_n(\theta)), \\ q_n(\theta) \to y(\theta) \quad a.e. \end{cases}$$

We know that for all $\theta' \in \Theta$ and all n,

$$V_n(\theta') \geq V_n(\theta) + h(\theta', q_n(\theta)) - h(\theta, q_n(\theta)).$$

In the limit, we obtain

$$\bar{V}(\theta') \geq \bar{V}(\theta) + h(\theta', y(\theta)) - h(\theta, y(\theta)).$$

The above equation means that $y(\theta) \in \partial^h \bar{V}(\theta)$.
Then, we get

$$\alpha(\theta) = -h(\theta, y(\theta)) + C(y(\theta)) \geq -h(\theta, \bar{q}(\theta)) + C(\bar{q}(\theta)). \tag{2.9}$$

Therefore, we have

$$\begin{aligned}\inf(PA') &= \liminf_n \int_\Theta [V_n(\theta) - h(\theta, q_n(\theta)) + C(q_n(\theta))] f(\theta) d\theta \\ &\geq \int_\Theta [\bar{V}(\theta) - h(\theta, \bar{q}(\theta)) + C(\bar{q}(\theta))] f(\theta) d\theta \\ &= J(\bar{V}, \bar{q})\end{aligned}$$

where in the above inequality we used equations (2.7), (2.8) and (2.9).
This shows that $(\bar{V}, \bar{q})$ is a solution of (PA'). □

2.4. **Existence of solutions for the general cost functions.** In this section, we extend Carlier's proof [4] to our model for the general cost functions. Suppose h satisfies (H1)-(H3) and (H5). We recall the minimization problem (PA'):

$$(PA')\begin{cases} \min\limits_{q(\theta),V(\theta)} & \int_\Theta \phi(\theta, V(\theta), q(\theta))d\theta \\ \text{s.t.} & V \text{ is h-convex,} \\ & q(\theta) \in \partial^h V(\theta), \\ & V(\theta) \geq 0 \end{cases}$$

where $\phi(\theta, V(\theta), q(\theta)) = [V(\theta) - h(\theta, q(\theta)) + C(q(\theta))]f(\theta)$. To prove the extension of the previous result regarding existence of solutions, we generalize (H4) to a larger class of cost functions and we further add an hypothesis (H6):

H4′. There exist $\alpha \leq 1$, $a > 0$ and $b \in \mathbb{R}$, such that for all $(\theta, q) \in \Theta \times \mathbb{R}^m_+$

$$h(\theta, q) \leq a\|q\|^\alpha - b.$$

H6. $\phi(\cdot,\cdot,\cdot)$ is a normal integrand, which means that for almost every $\theta \in \Theta$, $\phi(\theta,\cdot,\cdot)$ is lower semi-continuous and that there exists a borelian map $\bar{\phi}$ such that $\phi(\theta,\cdot,\cdot) = \bar{\phi}(\theta,\cdot,\cdot)$ for almost every $\theta \in \Theta$. There exist $A > 0$ and $\Psi \in L^1(\Theta)$ such that for almost every $\theta \in \Theta$ and every $(V, q) \in \mathbb{R} \times \mathbb{R}^m_+$,

$$\begin{aligned} \phi(\theta, V, q) &= V - h(\theta, q) + C(q) \\ &\geq A(|V| + \|q\|^\gamma) + \Psi(\theta), \quad \gamma \geq 1. \end{aligned}$$

Theorem 2. *Problem (PA') admits at least one solution.*

Proof. The proof is similar to that of Theorem 1. Indeed, according to (H6), we know that (V_n) is bounded in $W^{1,1}(\Theta)$. Using Proposition 7 we have:

$$\begin{cases} V_n \text{ converges to } \bar{V} \text{ in } L^1(\Theta) \text{ and uniformly on compact subsets of } \Theta, \\ \nabla V_n \text{ converges almost everywhere in } \Theta \text{ to } \nabla \bar{V}, \end{cases}$$

where $\bar{V}$ is h-convex and belongs to $W^{1,1}(\Theta)$.
Following the proof of Theorem 1, we can choose q_n to be measurable and such that for all $\theta \in \Theta$,

$$q_n(\theta) \in \operatorname*{argmin}_{q \in \partial^h V_n(\theta)} \{V_n(\theta) - h(\theta, q) + C(q)\}.$$

We can now define $\bar{q}$ as the measurable selection of set-valued maps

$$\theta \to \operatorname*{argmin}_{q \in \partial^h \bar{V}_n(\theta)} \{\bar{V}_n(\theta) - h(\theta, q) + C(q)\}.$$

Lastly, if y is a cluster point of a sequence of elements of $\partial^h V_n(\theta)$, then $y \in \partial^h \bar{V}(\theta)$. This enables us to prove that $(\bar{V}, \bar{x})$ is a solution using Fatou's Lemma. □

Note that there is a large class of cost functions $\phi(\theta, V(\theta), q(\theta)) = [V(\theta) - h(\theta, q(\theta)) + C(q(\theta))]f(\theta)$ (e.g., polynomials) satisfying the conditions imposed in (H4′) and (H6).

3. **Discrete problem.** Since Θ is an open bounded convex subset of $\mathbb{R}^p$, then the continuous problem (1) can be discretized in p dimensions as follows:

$$
\begin{array}{lll}
(PD) \ \max\limits_{q_i,t_i} & \sum\limits_{i_p=1}^{m_p} \cdots \sum\limits_{i_1=1}^{m_1} (t_{i_1,\dots i_p} - C(q_{i_1,\dots i_p})) f_{i_1,\dots i_p} \Delta_{i_1} \dots \Delta_{i_p} & \\
\text{s.t.} & h(\theta_{i_1,\dots i_p}, q_{i_1,\dots i_p}) - t_{i_1,\dots i_p} \geq 0, & i = 1,2,\dots,p \ \text{(IR)} \\
 & h(\theta_{i_1,\dots i_p}, q_{i_1,\dots i_p}) - t_{i_1,\dots i_p} \geq h(\theta_{i_1,\dots i_p}, q_{j_1,\dots,j_p}) - t_{j_1,\dots,j_p}, & \\
 & & \forall i,j = 1,2,\dots,p \ \text{(IC)}
\end{array}
$$

where

$$
\begin{aligned}
t_{i_1,\dots,i_p} &= t\left(\theta^1_{i_1}, \dots, \theta^n_{i_p}\right), \\
q_{i_1,\dots,i_p} &= q\left(\theta^1_{i_1}, \dots, \theta^n_{i_p}\right), \\
f_{i_1,\dots,i_p} &= f\left(\theta^1_{i_1}, \dots, \theta^n_{i_p}\right),
\end{aligned}
$$

in which $\theta^j_{i_j}$ represents the ij-th point in the j-th coordinate. The m_j denotes the total subintervals in the j-th coordinate and $\Delta_{i_j} = \theta^j_{i_j} - \theta^j_{i_j-1}$.

Existence Result for the Discrete Problem

To prove the existence result for the discrete problem we need to keep (H1), (H4′) and slightly modify (H6):

H6′. $\phi(\cdot,\cdot)$ is a normal integrand and there exist $A > 0$ and $\Psi \in L^1(\Theta)$ such that for almost every $\theta \in \Theta$

$$
\begin{aligned}
\phi(\theta, q) &= -h(\theta, q) + C(q) \\
&\geq A\|q\|^\gamma + \Psi(\theta), \quad \gamma \geq 1.
\end{aligned}
$$

Under the assumptions imposed above, the following existence result holds.

Theorem 3. *The discrete problem* (PD) *has at least one solution.*

Proof. From the (IR) constraint, we have

$$t_{i_1,\dots i_p} \leq h(\theta_{i_1,\dots i_p}, q_{i_1,\dots i_p}), \tag{3.1}$$

and then

$$t_{i_1,\dots i_p} - C(q_{i_1,\dots i_p}) \leq h(\theta_{i_1,\dots i_p}, q_{i_1,\dots i_p}) - C(q_{i_1,\dots i_p}). \tag{3.2}$$

Recall by (H6′) that, we have for fixed value of $\theta \in \Theta$, if $q \to \infty$, then $-h(\theta, q) + C(q) \to +\infty$. As a result, $h(\theta, q) - C(q) \to -\infty$. As a result, by (3.2), $t_{i_1,\dots i_p} - C(q_{i_1,\dots i_p}) \to -\infty$ which does not affect our maximization problem. This implies $q_{i_1,\dots i_p}$ is bounded. So is $h(\theta_{i_1,\dots i_p}, q_{i_1,\dots i_p})$ using (H4′). This means that $t_{i_1,\dots i_p}$ is bounded by (3.1). Therefore, we are searching for a maximum of the upper semi-continuous function on a compact set which definitely exists (Weiestrass-Lebesgue Lemma). □

4. **Solutions without the Spence Mirrlees condition.** In this section we study the discrete principal-agent problem (2). We first recall the following definition.

Definition 6. A function $h(\theta, q)$ satisfies the discrete Spence Mirrles condition if

$$h(\theta', q') - h(\theta, q') > h(\theta', q) - h(\theta, q) \qquad \forall \theta' > \theta, q' > q.$$

As we discussed in the introduction, the Spence Mirrlees condition plays an important role in solving the principal-agent problem. Without the Spence Mirrlees condition, there is no way to characterize the analytic solutions of the general problem (2); so numerical algorithms may be required to solve for an approximate solution. To characterize a local optimal solution as a stationary point of the problem, one needs to verify certain constraint qualifications. The commonly used constraint qualifications are the linear independence and the Mangasarian Fromovitz constraint qualification. The aim of this section is to find some sufficient condition for these constraint qualifications.

4.1. **Linear independence constraint qualification (LICQ).** Recall that a feasible solution of an optimization problem with inequality constraints satifies the LICQ if the gradients of all active constraints are linearly independent. The difficulty in verifying LICQ is that we are not able to say which of our constraints are active and which are inactive at the optimum. We have to consider the worst case where all of the inequality constraints are active at the optimum. It is clear that any subset of the linearly independent vectors is itself linearly independent. This means we are looking for sufficient conditions for LICQ.

Theorem 4. *LICQ holds for $n = 2$ if,*

$$\begin{cases} \frac{\partial h}{\partial q}(\theta_1, q_1) & \neq \frac{\partial h}{\partial q}(\theta_2, q_1); \\ \textit{and} & \\ \frac{\partial h}{\partial q}(\theta_2, q_2) & \neq \frac{\partial h}{\partial q}(\theta_1, q_2). \end{cases}$$

Proof. When we have only 2 customers, the constraints of the problem are:

$$\begin{aligned} h(\theta_1, q_1) - t_1 &\geq 0, \\ h(\theta_2, q_2) - t_2 &\geq 0, \\ h(\theta_1, q_1) - t_1 &\geq h(\theta_1, q_2) - t_2, \\ h(\theta_2, q_2) - t_2 &\geq h(\theta_2, q_1) - t_1. \end{aligned}$$

Suppose that all of our constraints are active at optimum. Then the linear combination of the gradient vectors gives four equations with four unknowns, $\lambda_1, \lambda_2, \lambda_3, \lambda_4$,

$$\begin{cases} -\lambda_1 \frac{\partial h}{\partial q}(\theta_1, q_1) - \lambda_3 \frac{\partial h}{\partial q}(\theta_1, q_1) + \lambda_4 \frac{\partial h}{\partial q}(\theta_2, q_1) = 0, \\ -\lambda_2 \frac{\partial h}{\partial q}(\theta_2, q_2) + \lambda_3 \frac{\partial h}{\partial q}(\theta_1, q_2) - \lambda_4 \frac{\partial h}{\partial q}(\theta_2, q_2) = 0, \\ \lambda_1 + \lambda_3 - \lambda_4 = 0, \\ \lambda_2 - \lambda_3 + \lambda_4 = 0. \end{cases}$$

Finding λ_1 and λ_2 from the last two equations and substituting them back into the rest of the equations yields

$$\begin{cases} -\lambda_4 \frac{\partial h}{\partial q}(\theta_1, q_1) + \lambda_4 \frac{\partial h}{\partial q}(\theta_2, q_1) = 0, \\ -\lambda_3 \frac{\partial h}{\partial q}(\theta_2, q_2) + \lambda_3 \frac{\partial h}{\partial q}(\theta_1, q_2) = 0. \end{cases}$$

By the above equations, it is obvious that if

$$\begin{cases} \frac{\partial h}{\partial q}(\theta_1, q_1) & \neq \frac{\partial h}{\partial q}(\theta_2, q_1); \\ \text{and} & \\ \frac{\partial h}{\partial q}(\theta_2, q_2) & \neq \frac{\partial h}{\partial q}(\theta_1, q_2) \end{cases}$$

then all the $\lambda_i, i = 1, 2, 3, 4$ have to be zero. This implies that LICQ is satisfied. $\square$

The theorem below shows the weakness of LICQ for dealing with the principal-agent problem without the Spence Mirrlees condition in general.

Theorem 5. *LICQ is never satisfied for $n \geq 3$ when all of our constraints are active.*

Proof. Suppose λ_i are the coefficients of the gradient vectors. Then LICQ holds if

$$\sum_{i=1}^{n^2} \lambda_i \begin{pmatrix} \frac{\partial W_i}{\partial q_1} \\ \vdots \\ \frac{\partial W_i}{\partial q_n} \\ ---- \\ \frac{\partial W_i}{\partial t_1} \\ \vdots \\ \frac{\partial W_i}{\partial t_n} \end{pmatrix} = 0,$$

implies all λ_i are zero where W_i represents the i-th constraint. The number of λ_is is equal to the number of constraints, which is n^2. On the other hand, the number of equations is equal to the number of components of each gradient matrix, which is $2n$. For $n \geq 3$ the number of λ_is is larger than the number of equations. Hence, λ_is are always dependent. □

4.2. **Mangasarian-Fromovitz constraint qualification (MFCQ).** First, recall that the Mangasarian-Fromovitz Constraint Qualification (MFCQ) holds at a feasible solution of an optimization problem with inequality constraints if and only if all gradient of the active constraints are positively linearly independent. We assume the worst case scenario, which is the case in which all of the inequality constraints are active at the optimum. If only a subset of the inequality constraints were active, then our sufficient condition would still make sure that the vectors corresponding to these quality constraints are positively linearly independent because the set of vectors would be a subset of all the vectors that we prove to be positively linearly independent under our sufficient condition.

Definition 7. (Principal-Agent Problem Constraint Qualification)
Principal-Agent Problem Constraint Qualification (PAPCQ) holds at a certain point if for each fixed value of j we have,

$$\frac{\partial h}{\partial q}(\theta_i, q_j) > \frac{\partial h}{\partial q}(\theta_j, q_j), \qquad \forall i \neq j.$$

Then we have the following theorem.

Theorem 6. *MFCQ holds if PAPCQ holds.*

Proof. Suppose λ_i is the multiplier of the (IR) constraint associated with the customer with type θ_i and λ_{ij} is the multiplier of the (IC) constraint associated with the customer with type θ_i who likes to hide his type as θ_j. In the general case of having n taste-type customers we deal with the following gradient vectors

$$\sum_{i=1}^{n}\lambda_i \begin{pmatrix} \mathbf{0}\\ -\frac{\partial h}{\partial q}(\theta_i,q_i)\\ \mathbf{0}\\ ----\\ \mathbf{0}\\ 1\\ \mathbf{0}\end{pmatrix}\begin{array}{l} \\ i\text{-th row}\\ \\ \\ \\ i\text{-th row}\\ \\ \end{array} + \sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_{ij}\begin{pmatrix} \mathbf{0}\\ -\frac{\partial h}{\partial q}(\theta_i,q_i)\\ \frac{\partial h}{\partial q}(\theta_i,q_j)\\ \mathbf{0}\\ ----\\ \mathbf{0}\\ 1\\ -1\\ \mathbf{0}\end{pmatrix}\begin{array}{l} \\ i\text{-th row}\\ j\text{-th row}\\ \\ \\ \\ i\text{-th row}\\ j\text{-th row}\\ \\ \end{array} = 0.$$

where the horizontal line $(----)$ divides the matrix into two parts. The first part is the gradient with respect to q and the second part is the gradient with respect to t. The $\mathbf{0}$ denotes that in each part of the matrix every entry is zero except the i-th and j-th rows.
This implies the set of equations:

$$\begin{cases} -\lambda_i\frac{\partial h}{\partial q}(\theta_i,q_i) - \sum_{j=1,j\neq i}^{n}\lambda_{ij}\frac{\partial h}{\partial q}(\theta_i,q_i) + \sum_{j=1,j\neq i}^{n}\lambda_{ji}\frac{\partial h}{\partial q}(\theta_j,q_i) = 0, & \forall 1\le i\le n\\ \lambda_i + \sum_{j=1}^{n}\lambda_{ij} - \sum_{j=1}^{n}\lambda_{ji} = 0. \quad \forall 1\le i\le n \end{cases}$$

From the last equation we know that $\lambda_i = -\sum_{j=1}^{n}\lambda_{ij} + \sum_{j=1}^{n}\lambda_{ji}$ for all i. Substituting in the first equation yields

$$\sum_{j=1}^{k}\lambda_{ji}\left(-\frac{\partial h}{\partial q}(\theta_j,q_i) + \frac{\partial h}{\partial q}(\theta_i,q_i)\right) = 0, \qquad \forall i.$$

Hence, if all $\left(-\frac{\partial h}{\partial q}(\theta_j,q_i) + \frac{\partial h}{\partial q}(\theta_i,q_i)\right)$ have the same sign then all the λ_{ji}s will be forced to be zero. Thus, we will have MFCQ. □

If the problem satisfies the PAPCQ then we are able to write the Karush-Kuhn-Tucker condition for the problem. If the problem is concave, the Karush-Kuhn-Tucker condition is necessary and sufficient for optimality. Otherwise it just gives us a set of stationary points that the optimal solution will belong to.
Example. There is a large class of functions $h(\theta,q)$ that do not satisfy the Spence Mirrlees Condition. However, they satisfy the PAPCQ at a certain point. In the following we will see one of those examples. Suppose we have three customers.
Define,

$$C(q) = \left|(q-0)\left(q-\frac{5\pi}{4}\right)\left(q-\frac{7\pi}{4}\right)\right|,$$

and

$$h(\theta,q) = \begin{cases} \cos(q) & \theta=\theta_1\\ \cos\left(\frac{15}{4}q - \frac{43\pi}{16}\right) & \theta=\theta_2\\ \cos\left(\frac{10}{7}q + \frac{3\pi}{2}\right) & \theta=\theta_3. \end{cases}$$

We claim that the optimal point is $(q_1^*,q_2^*,q_3^*,t_1^*,t_2^*,t_3^*) = (0,\frac{5\pi}{4},\frac{7\pi}{4},1,1,1)$, since at this point the negative part of the objective function, $C(q)$, will disappear and the positive term, t, achieves its maximum value; using the fact that $h(\theta,q)$ is an upper bound for t. In other words, $\cos(\cdot)$ is the upper bound for t. So, the maximum value that t can achieve is 1. This point also satisfies all of our constraints. This means (q^*,t^*) is feasible. Hence, this is a real optimal solution for this problem.

The function $h(\theta, q)$ does not satisfy the Spence Mirrlees Condition in the discrete term. To see the point, without loss of generality suppose $q_1 > q_3$. This case is impossible since $h(\theta_3, q_1) - h(\theta_1, q_1) = -1$ and $h(\theta_3, q_3) - h(\theta_1, q_3) = 1 - 0.7 = 0.3$. Now, assume $q_1 < q_3$. This case is also impossible since, $h(\theta_3, q_3) - h(\theta_2, q_3) = 0.08$ and $h(\theta_3, q_1) - h(\theta_2, q_1) = 0.5$.

It is also possible to make the function $h(\theta, q)$ continuous such that the Spence Mirrlees Condition fails. To do that, we need to define a small enough neighborhood around each point θ_1, θ_2 and θ_3 such that their pairwise intersection is empty. Then, the function $h(\theta, q)$ that we already defined can be used in each small neighborhood. Any other function can be defined between those neighborhoods to make the whole function continuous. In that case, since on a small enough neighborhood for each of θ_1, θ_2 and θ_3 the satisfaction of the customer is just a function of q, SMC will fail at θ_1, θ_2 and θ_3.
Although this function does not satisfy the Spence Mirrlees Condition, it satisfies the PAPCQ. This means, for all i and j, we have

$$\frac{\partial h}{\partial q}(\theta_i, q_j) > \frac{\partial h}{\partial q}(\theta_j, q_j)$$

which is the PAPCQ condition. So, the problem satisfies the MFCQ constraint qualification at the optimum point. This example can be generalized to any number of customers.

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