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QUANTUM ERROR CORRECTION FOR DIAGONAL CHANNELS

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Abstract. The properties of quantum channels with diagonal Kraus operators are examined. These are exactly the channels which allow full transmission of classical information.

The condensed matrix is presented. The quantum channel has capacity zero iff the condensed matrix has full rank, otherwise it can transmit at least 1 qubit per use.

A tight bound on the number of error operators a random quantum channel may have and still be correctable is given for both real and complex diagonal channels.

1. Introduction.

1.1. Diagonal Quantum Channels.

Quantum channels are mathematical representations of operations performed on qubits. Usually, these operations are undesirable, and the goal is to correct their effects. For an overview of quantum computing and quantum error correction, see [1] or [2]. For the purposes of error correction, a quantum state is represented as a square complex matrix ρ (a *density matrix*). This gives all the information about the current interactions of a given set of qubits. Specifically, when measuring a quantum state, we extract one basis vector from the orthonormal basis set $\{|\varphi_i\rangle \,|i = 0, 1, ..n\}$[1]. Given any density matrix, $\langle\varphi_i| \rho |\varphi_i\rangle$ gives the probability of the measurement outputting the i^{th} vector of the basis.

A quantum channel is the most general possible type of operation on a set of qubits (or on a density matrix). Quantum algorithms, for example, can be thought of as specific quantum channels. Transmission of entangled photons through a specific fibre-optic cable could also be represented as a quantum channel. It is obvious, then, that quantum channels can be combined in various ways (e.g., an algorithm deciding which of two fiber-optic cables to use to send 5 entangled photons from point A to point B, based on input from a sixth photon, the entire process could be represented as a single quantum channel, with a 2^6 x 2^6 input density matrix and a 2^5 x 2^5 output density matrix).

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[1]The vector is expressed in bra-ket notation, which will be used extensively in this document. Explanations of the notation are widely available.

Without loss, I will only consider quantum channels have the same dimension input and output, which can be obtained by adding qubits to one end of the quantum channel. For example, in the above example, a $|0\rangle$ qubit could be added to the output to make a quantum channel preserving the dimension of ρ.

Definition 1. A general quantum channel is a completely positive, trace preserving linear map [1]. This means that it is a map $\Phi : \mathbb{C}^{n \times n} \to \mathbb{C}^{m \times m}$, such that $\mathrm{Tr}(\Phi(A)) = \mathrm{Tr}(A)$, and if A is Hermitian with all positive eigenvalues so is $\Phi(A)$. Without loss, we consider for quantum channels only the case where $m = n$ (as extra independent qubits can be added to the channel at either end, increasing the dimension without changing the information transmission properties). By Choi's Thereom [3], all maps of this type can be represented as:

$$\mathcal{E}(\rho) = \sum_a E_a \rho E_a^\dagger \tag{1}$$

(The E_a are complex matrices). The E_a are called *Kraus operators*, or *error operators.* In the cases where the dimension of ρ is preserved, they will be square. Again by Choi's Theorem, the quantum channels (completely positive, trace preserving maps) are exactly the cases where:

$$\sum_a E_a^\dagger E_a = \mathbb{I} \tag{2}$$

It is convenient to notice that sending qubits through two channels simultaneously can be represented by a new, larger channel. For qubits not entangled across the two channels, this is expressed (using the Kronecker product) as:

$$\mathcal{E}_1(\rho_1) = \sum_a E_a \rho_1 E_a^\dagger \tag{3}$$

$$\mathcal{E}_2(\rho_2) = \sum_a F_a \rho_2 F_a^\dagger \tag{4}$$

$$\mathcal{E}_3(\rho_1 \otimes \rho_2) = \sum_a \sum_b (E_a \otimes F_b)(\rho_1 \otimes \rho_2)(E_a \otimes F_b)^\dagger \tag{5}$$

This can easily be extended to the non-separable density matrices by linearity.

In this paper, I am interested in channels which exactly retain all classical information in the standard basis. This corresponds to the channels which are not only trace preserving, but leave the diagonal of all the positive, trace 1 density matrices (and by linearity the diagonal of all matrices) unchanged.

Theorem 1. *The quantum channels which retain all classical information unchanged are precisely those with diagonal error operators.*

Proof. We see that all permissible quantum channels with diagonal error operators leave the diagonals of the density matrix unchanged when we consider that for any diagonal channel and any i, $\sum_a |(E_a)_{ii}|^2 = 1$. Doing the quantum channel operation for general diagonal matrices with this constraint and general ρ immediately shows the result.

The other direction is somewhat trickier. Take $\langle \varphi_a| = \sum_i b_{i,a} \langle i|$ to be the j^{th} row of E_a. Take ρ_1 to be a diagonal n by n matrix, with $\frac{1}{n-1}$ being every entry on the diagonal except the j^{th} entry, which is zero. This satisfies $\mathrm{Tr}(\rho_1) = 1$.

Note that, for the j^{th} element of the diagonal to be preserved, we have to have

$$\sum_a \langle\varphi_a|\,\rho_1\,|\varphi_a\rangle = 0 \tag{6}$$

Expanding and simplifying, we have

$$\sum_a \sum_{i\neq j} |b_{i,a}|^2 = 0 \tag{7}$$

So $i \neq j \implies b_{i,a} = 0$. □

1.2. **Correctable Codes.** For the purposes of this paper, I am only interested in whether any information at all can be passed through a quantum channel, not how much. Therefore, I consider a quantum channel to be correctable if at least 1 general qubit can be passed through it exactly.

Definition 2. For the purposes of this paper, *correctable diagonal channels* are those that can transmit at least 1 qubit of quantum information exactly.

Theorem 7 shows that it does not give an uncorrectable diagonal channel extra power to be used multiple times with entangled input. This is not necessarily true for general quantum channels.

1.3. **Condensed Vector Set and Condensed Matrix.**

Definition 3. Each n-error, m-dimensional quantum channel has a condensed vector set, represented as V, with dimension m and size n. A subscript vector of the set in this scheme ($\vec{v}_k$) represents the column vector associated with the k^{th} error operator, E_k.

The condensed vector set is useful because it provides a succint way to represent the channel.

Definition 4. Define a pointwise conjugate product of two vectors, $\vec{u}$ and $\vec{v}$, to be $\vec{u} \times \vec{v} = \vec{w}$, defined by $w_j = \bar{u}_j v_j$.

Definition 5. Each m-dimensional quantum channel has a condensed matrix, represented as W. To define it, take $\vec{w}_i$ to be the i^{th} column of the matrix. n is the size of the condensed vector set V. W is then:

$$\vec{w}_{(i-1)n+j} = \vec{v}_i \times \vec{v}_j \tag{8}$$

This matrix has m^2 columns and n rows.

The condensed matrix's rank is usefull in determining whether a diagonal channel is correctable, and if so, the matrix itself can be used to determine what the correction code is.

The condensed matrix also has another interesting property.

Lemma 1. *Given two diagonal quantum channels with condensed vector sets V_1 and V_2, define the condensed vector set of the quantum channel formed by entangling input across the two original channels as V_3. Let W_1, W_2, and W_3 be the corresponding condensed matrices. Given these conditions, W_3 is a formed by a permutation of the columns of $W_1 \otimes W_2$.*

Proof. Assume that the vectors in V_1 have length m, and that the vectors in V_2 have length n. Impose some ordering on V_1 and V_2. Define $\vec{a}_i$ as the row vector obtained by taking the i^{th} element of each vector in V_1 in order. Similarly define $\vec{b}_j$ as the equivalent row vector for j^{th} row of V_2.

Note that the $[(i-1)m+j-1]^{th}$ row of V_3 (for simplicity, denoted $\vec{c}$) is $\vec{c} = \vec{a}_i \otimes \vec{b}_j$. Therefore, the same row of W_3 is

$$\vec{c} \otimes \overline{\vec{c}} = (\vec{a}_i \otimes \vec{b}_j) \otimes \overline{(\vec{a}_i \otimes \vec{b}_j)} = \vec{a}_i \otimes \vec{b}_j \otimes \overline{\vec{a}_i} \otimes \overline{\vec{b}_j} \tag{9}$$

Note that the $[(i-1)m+j-1]^{th}$ row of $W_1 \otimes W_2$, denoted $\vec{d}$ for convenience, is

$$\vec{d} = W_{1i} \otimes W_{2j} = (\vec{a}_i \otimes \overline{\vec{a}_i}) \otimes (\vec{b}_j \otimes \overline{\vec{b}_j}) \tag{10}$$

The lemma is now easy to see. □

1.4. **Conjugate-Closed Vector Sets.**

Definition 6. A conjugate-closed vector set V has the property that $\vec{v} \in V \implies \bar{\vec{v}} \in V$, where $\bar{\vec{v}}$ is the vector obtained by taking the complex conjugate of every entry in $\vec{v}$.

Theorem 2. *The span of any conjugate-closed vector set has a basis of real vectors.*

Proof. Take any pair of conjugate vectors in the set. Add them together and divide by 2 to get the real part. Subtract them and multiply by $\frac{i}{2}$ to get the imaginary part. The span of these two real vectors cannot be more than the span of the space (after all, they are in the space). Their span cannot be less, as the conjugate pair is in their span. So it is the same space. Repeat for every conjugate pair to find a real vector set spanning the space. Reduce the resulting real vector set to a real basis. □

By theorem 2, the condensed matrix for any diagonal channel has a real basis for its column space, and therefore also its left null space.

2. **Main Results.**

2.1. **Diagonal Channel Correction and the Condensed Matrix.**

Theorem 3. *(Due to Knill and Laflamme* [4]*). Necessary and sufficient conditions for recovery of a state are non-zero logical encoded states $|0_L\rangle$ and $|1_L\rangle$ such that for all the error operators E_c,*

$$\langle 0_L| E_c^\dagger E_d |1_L\rangle = 0 \tag{11}$$

$$\langle 0_L| E_c^\dagger E_d |0_L\rangle = \langle 1_L| E_c^\dagger E_d |1_L\rangle \tag{12}$$

[4] explicitly shows how to correct errors given two states satisfying these conditions.

We can represent these two new basis states as

$$|0_L\rangle = \sum_x a_x |x\rangle \tag{13}$$

$$|1_L\rangle = \sum_x b_x |x\rangle \tag{14}$$

Theorem 4. *Assume an m-dimensional diagonal quantum channel. It is uncorrectable if and only if the rank of its condensed matrix is exactly m.*

Proof. Take the diagonal entries of E_i, $(E_i)_{jj}$, to be $E_{i,j}$ for conciseness. The constraints become:

$$\sum_j \overline{a_j (E_{c,j})} E_{d,j} b_j = 0 \tag{15}$$

$$\sum_j \overline{a_j (E_{c,j})} E_{d,j} a_j = \sum_j \overline{b_j (E_{c,j})} E_{d,j} b_j \tag{16}$$

It is easy to show that the trace-preserving property of the E_i makes the above constrain the $|0_L\rangle$ and $|1_L\rangle$ to have equal magnitude and be perpedicular. The above is trivially satisfied for two vectors of zero magnitude, but a non-zero solution is required.

Define a vector set Q to be $Q = \{\vec{q} : (\exists c, d)(\forall j)(q_j = \overline{(E_{c,j})} E_{d,j})\}$. It is easy to see that the vectors in Q are exactly the columns of the condensed matrix for the channel. It is also easy to see that if we define two vectors, $\vec{x}$ and $\vec{y}$, such that $x_j = \bar{a}_j b_j$ and $y_j = |a_j|^2 - |b_j|^2$, the channel is correctable iff these vectors are in the left null space of the condensed matrix. Therefore, it is necessary that the left null space contain non-zero vectors, which only occurs if the rank of the condensed matrix is less than m.

Assume that there exists a non-zero vector $\vec{d}$ in the left null space. Without loss of generality, assume that it is real, by theorem 2. Also, assume that $|0_L\rangle$ and $|1_L\rangle$ are real (because we just need a single solution for them). To solve for values of $|0_L\rangle$ and $|1_L\rangle$ satifying the constraints, expand $|0_L\rangle$ to a diagonal matrix A, $|1_L\rangle$ to a diagonal matrix B, and $\vec{d}$ to a diagonal matrix D. By expand, what is meant is to use the i^{th} vector entry as the i^{th} component of the matrix, diagonally. In the following equations, c_1 and c_2 are just arbitrary real scaling coefficients. The equations above imply:

$$c_1 D = AB \tag{17}$$

$$c_2 D = A^2 - B^2 \tag{18}$$

For any value of the free parameters c_1 and c_2, this is satisfied by:

$$A = \sqrt{\frac{c_2 D + \sqrt{c_2^2 D^2 + 4c_1^2 D^2}}{2}} \tag{19}$$

$$B = \sqrt{\frac{-c_2 D + \sqrt{c_2^2 D^2 + 4c_1^2 D^2}}{2}} \tag{20}$$

Above, all square roots are principal.

This satisfies all the original conditions, and so the two vectors must have equal magnitudes for any choice of c_1 and c_2. Choose c_1 and c_2 so that one vector has unit norm, and the other must. $\square$

This theorem has two immediate corollaries.

Corollary 1. *Any real diagonal quantum channel where the state space is m-dimensional and there are n error operators, with $m > \binom{n}{2} + \binom{n}{1} = \frac{n^2+n}{2}$, is correctable for at least 1 qubit.*

Proof. The condensed matrix for real channels has maximum rank $\binom{n}{2} + \binom{n}{1}$, because the pointwise conjugate product is commutative for real vectors. $\square$

Corollary 2. *Any (in general, complex) diagonal quantum channel where the state space is m-dimensional and there are n error operators, with $m > n^2$, is correctable.*

Proof. The condensed matrix for all diagonal channels has maximum rank n^2. $\square$

2.2. **Tight Bounds for Number of Error Operators.** The bounds given in corollaries 1 and 2 are tight in the sense that for random channels with any more than the given number of error operators, diagonal quantum channels are uncorrectable with probability 1. In practice, it appears that if these channels are chosen approximately randomly, when the bounds are exceeded the channel is always uncorrectable.

Lemma 2. *For any given non-zero real symmetric A, a vector with real entries selected uniformly at random $\vec{r} \in \mathbb{R}^m$ has the property $\vec{r}^T A\vec{r} \neq 0$ with probability 1.*

This is easily seen by, for any A, working in the basis where it is diagonal.

Lemma 3. *For any m, a randomly chosen set of up to $\frac{m^2+m}{2}$ vectors V in $\mathbb{R}^m$ has the property that with probability 1, $Q = \{\vec{v} \otimes \vec{v} : \vec{v} \in V\}$ is linearly independent.*

Proof. Note that $\vec{v} \otimes \vec{v}$ has $v_{im+j} = v_{jm+i}$, so it has at most $\frac{m^2+m}{2}$ free variables. Take the $\frac{m^2+m}{2}$-dimensional vector without the doubled terms, and call it $\vec{r}$. It is sufficient to prove that with random $\vec{v}$, $\vec{r}$ is not perpendicular to any given non-zero $\vec{u}$. Then, to construct the linearly independent set needed, it is sufficient to pick a vector from the space orthogonal to the current vector set's span and note that the next vector in the set will almost certainly have a non-zero projection onto the orthogonal space.

Take any m by m set of numbers with $u_{ij} = u_{ji}$, and write:

$$\begin{aligned} \sum_{i=1}^{m}\sum_{j=i}^{m} u_{ij}v_iv_j &= \sum_{i=1}^{m}\sum_{j=1}^{m} \frac{(1+\delta_{ij})u_{ij}}{2}v_iv_j \\ &= \vec{v}^T S\vec{v} \end{aligned} \tag{21}$$

The question is solved by an appeal to lemma 2. $\square$

Theorem 5. *A random real diagonal channel with m dimensions, n error operators, and $m \leq \frac{n^2+n}{2}$ is non-correctable with probability 1.*

Proof. Consider the rows of condensed matrix for the channel, which by lemma 3 are almost certainly linearly independent. The rows are normalized, but since any $\vec{v}$ satisfying $\vec{v}^T S\vec{v} = 0$ implies all its multiples do as well, all the probability results hold when we restrict the result to random unit vectors. The condensed matrix therefore has rank m and therefore uncorrectable by theorem 4 almost certainly. $\square$

A similar result can be shown for complex channels. The proof begins with a result similar to lemma 2, but this one is less obvious.

Lemma 4. *Take any non-zero m by m complex matrix A, and any vector $\vec{v}$. Call $\vec{v}^\dagger A\vec{v} = 0$ property P. Then, any A with random unit $\vec{v}$ has property P with probability zero.*

Proof. Note that the property P is invariant under multiplication of the vector by a real scalar, and so the lemma is equivalent to the same assertion for all random (non-unit) vectors $\vec{v}$. Further note that the lemma is trivially true unless there are at least m linearly independent vectors satisfying P. I can assume I am working in the basis defined by these vectors, and so without loss of generality assume $a_{ii} = 0$.

Assume that there exists an A such that $(A, \vec{v})$ satisfies P for vectors chosen uniformly at random, with fixed, non-zero probability. For simplicity, assume that we choose random vectors by picking each of the entries in turn, and picking the magnitude at random from $\mathbb{R}$ and the argument at random from $[0, 2\pi]$.

Note that the property P is invariant under multiplication of $\vec{v}$ by a complex number of unit magnitude. Therefore, assume without loss of generality that the k^{th} entry in $\vec{v}$, v_k, is real (for a fixed k).

Now, we choose v_k uniformly at random in $\mathbb{R}$. There must be a probability $\epsilon > 0$ of choosing v_k such that $\vec{v}$ will still satisfy property P with probability at least $\delta > 0$. Otherwise, choosing a v_k such that $\vec{v}$ satisfies P with non-zero probability has zero probability, contradicting the original assumption that P is satisfied on the entire vector set with non-zero probability.

This implies that for an infinite number of distinct v_k, given the choice v_k for the k^{th} vector entry, $\vec{v}$ still satisfies P with probability at least δ. We can therefore pick a set $Q = \{v_{k,i} : 1 \leq i \leq \lceil 2\delta^{-1} \rceil, v_{k,i} = v_{k,j} \iff i = j\}$ that has the additional property that the associated vectors still satisfy P with probability at least δ. Define V_i to be the vector set satisfying P for each $v_{k,i}$, but without the k^{th} row of the vectors. Define $p(V)$, where V is a vector set, to be the probability that a random vector satisfies $\vec{v} \in V$. Note that we cannot have $p(V_i \cap V_j) = 0$ for all $i \neq j$, or the total probability is greater than 1:

$$\sum_i p(V_i) \geq \lceil 2\delta^{-1} \rceil \delta \geq 2 > 1 \tag{22}$$

Therefore, for the original assumption to be true there must be two distinct values of v_k, b and c, such that there is a non-zero probability of the rest of the vector satisfying the property P for both values of v_k simultaneously.

This property is best expressed in the summation expression as:

$$\begin{aligned} 0 &= \left(\sum_{i \neq k} \sum_{j \neq k} v_i^\dagger a_{ij} v_j \right) + \sum_i b v_i^\dagger a_{i,k} + \sum_i b a_{k,i} v_i \\ &= \left(\sum_{i \neq k} \sum_{j \neq k} v_i^\dagger a_{ij} v_j \right) + \sum_i c v_i^\dagger a_{i,k} + \sum_i c a_{k,i} v_i \end{aligned} \tag{23}$$

$$b \left(\sum_i v_i^\dagger a_{i,k} + \sum_i a_{k,i} v_i \right) = c \left(\sum_i v_i^\dagger a_{i,k} + \sum_i a_{k,i} v_i \right) \tag{24}$$

$$\implies \sum_{i \neq k} v_i^\dagger a_{i,k} + \sum_{i \neq k} a_{k,i} v_i = 0 \tag{25}$$

Let $\vec{v}' = \vec{x} + i\vec{y}$ be the vector $\vec{v}$ without the k^{th} row. Let $\vec{u}_1 = \vec{d} + i\vec{f}$ be the vector of $a_{k,i}$ without a_{ii} and let $\vec{u}_2 = \vec{g} + i\vec{h}$ be the vector of $a_{i,k}$ without a_{ii}. Splitting the real and imaginary parts of the equation, we have:

$$\vec{x}\vec{d} + \vec{x}\vec{g} - \vec{y}\vec{f} + \vec{y}\vec{h} = \vec{x}(\vec{d} + \vec{g}) + \vec{y}(\vec{h} - \vec{f}) = 0 \tag{26}$$

$$\vec{x}\vec{h} + \vec{x}\vec{f} - \vec{y}\vec{g} + \vec{y}\vec{d} = \vec{x}(\vec{h} + \vec{f}) + \vec{y}(\vec{d} - \vec{g}) = 0 \tag{27}$$

Note that $\vec{d}$,$\vec{g}$,$\vec{f}$, and $\vec{h}$ are fixed and arbitrary. Continuing, we write:

$$0 = \begin{bmatrix} \vec{x}^T & \vec{y}^T \end{bmatrix} \begin{bmatrix} \vec{d} + \vec{g} \\ \vec{h} - \vec{f} \end{bmatrix} = \begin{bmatrix} \vec{x}^T & \vec{y}^T \end{bmatrix} \begin{bmatrix} \vec{h} + \vec{f} \\ \vec{d} - \vec{g} \end{bmatrix} \tag{28}$$

By inspection, 28 is satisfied with non-zero probability only when $\vec{d} = \vec{g} = \vec{f} = \vec{h} = 0$. This is equally valid for every row of the matrix, so only the zero matrix satisfies the assumed conditions on A. □

Lemma 5. *For any perfect square m, a randomly chosen set of up to m^2 vectors V in $\mathbb{C}^m$ has the property that with probability 1, $Q = \{\vec{v} \otimes \vec{v} : \vec{v} \in V\}$ is linearly independent.*

Proof. Note that Lemma 4 is equivalent to saying there are no fixed non-zero vectors $\vec{u}$ such that there is a nonzero probability that a random vector $\vec{v}$ will have the property $(\vec{v} \otimes \bar{\vec{v}}) \cdot \vec{u} = 0$ (consider the entries of $\vec{u}$ as the entries of A). Use induction, noting that the next vector added Q is almost certainly not perpendicular to any given vector in the space orthogonal to the span of Q. □

Theorem 6. *A random diagonal channels with m dimensions, n error operators, and $m \leq n^2$ is non-correctable with probability 1.*

Proof. Turn the problem sideways, and consider the rows of the condensed matrix as vectors of the set Q from lemma 5, which are almost certainly linearly independent. There are m rows, which is less than n^2 columns. Therefore, the condensed matrix almost certainly has rank m, and is almost certainly not correctable. □

2.3. **Diagonal Channels are Weakly Additive.** In this paper, the main concern for quantum channels is determining whether any quantum information at all can be transmitted. Up to this point, it has been assumed that only one use of any given channel is permitted. The following theorem provides a weak additivity condition on the diagonal channels, which allows us to state that if a diagonal quantum channel cannot transmit useful quantum information with one use, its total capacity allowing multiple uses is also zero.

Theorem 7. *If a quantum channel is constructed by sending entangled states through two separate diagonal quantum channels, the resulting (separable) quantum channel is correctable iff at least one of the separate channels is.*

Proof. Obviously, if either of the original channels is correctable the resulting one will also be correctable. It is not obvious that any two non-correctable diagonal channels still are non-correctable when used together.

Lemma 1 gives the machinery required to prove theorem 7. Assume that the first channel is m-dimensional, and the second is n-dimensional. Recall from the previous sections that for the two initial channels to be non-correctable it is necessary that $\text{Rank}(W_1) = m$ and $\text{Rank}(W_2) = n$, and for the third channel to be non-correctable it is sufficient to have $\text{Rank}(W_3) = mn$. Remember that permutation of columns of a matrix does not affect its rank.

To clinch the proof, recall:

$$\text{Rank}(W_3) = \text{Rank}(W_1 \otimes W_2) = \text{Rank}(W_1)\,\text{Rank}(W_2) \tag{29}$$

Therefore, if the two initial channels are non-correctable, $\text{Rank}(W_3) = mn$, proving the theorem. □

Corollary 3. *Any diagonal channel that cannot transmit quantum information with a single use of the channel has total capacity zero, even over multiple uses of the channel with entangled input.*

3. Open Problems.

3.1. It would be interesting to know more about the set of other diagonal quantum channels (with more than the given number of error operators) that are correctable. We know that the set has measure zero in the full set of diagonal quantum channels, but we do not know much else.

3.2. General quantum channels need subchannels with correctable classical information before they have a hope of transmitting quantum information. It would be interesting to try to apply these results generally, by first trying to correct classical information and then determining whether the resulting diagonal sub-channels are correctable. Unfortunately, this approach is probably not very useful because most techniques for correcting classical information would destroy quantum information.

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