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STATISTICAL MODELS FOR THE BANKER'S OFFER IN DEAL OR NO DEAL

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Abstract. *Deal or No Deal* is a television game show that involves one contestant, one banker and 26 briefcases containing different dollar values ranging between one cent and one million dollars. The allocation of dollar amounts inside the briefcases is unknown prior to the game to both the contestant and the banker. The contestant selects one briefcase to start the game that remains closed until the end. The game is played to a maximum of nine rounds, with a certain number of briefcases opened each round, revealing the dollar amounts. After every round the banker will submit a dollar offer to the contestant in an attempt to buy the contestant's briefcase.

In this paper, the question of interest is how exactly the banker determines the offer for each of the nine rounds that will be given to the contestant. This paper will focus on the formulation of the banker's offers. The data set collected by playing the online National Broadcasting Corporation (NBC) version and by watching the television (NBC) version of the games how are both analyzed to develop and compare several candidate models for the banker's offers. These models will then be tested on new data points to determine how well the banker's offer can be predicted for the online and television versions of the game show *Deal or No Deal*.

KEY WORDS: Data analysis, Statistical modeling, Risk theory

1. **Introduction.**

Deal or No Deal is a popular television game show that was originally produced by the Netherlands Company, Endemol, in 2002 (Post, Baltussen and Van den Assem, 2006). Different versions of the game, including online games, have appeared in over 65 countries and 5 continents around the world (NBC, 2009). This paper will focus on the online (NBC) version and the American television version of the game, which is also produced by NBC.

Each game involves a contestant/player, a banker and 26 briefcases numbered 1 to 26, that contain 26 known dollar values between \$0.01 to \$1,000,000 randomly assigned to these briefcases (Post, Baltussen and Van den Assem, 2006). At the beginning of the game all 26 cases are closed so the contestant (and the banker) does not know the dollar amount in any particular briefcase. The contestant begins the game by selecting one of the 26 briefcases, that remains closed until the end of the game. The contestant then opens the remaining 25 briefcases by opening up 6, 5, 4, 3, 2, 1, 1, 1, 1 briefcases in the respective nine possible rounds. Once a briefcase is selected, the dollar amount in it is revealed to both the contestant and the banker. At the end of every round, the banker will submit a money offer to buy the contestant's briefcase, because there is a chance that the contestant's briefcase contains a large value (e.g., one million dollars). The player, after hearing the banker's offer, can either say "Deal", accepting the banker's offer and forfeiting the unknown value inside his briefcase or "No Deal", rejecting the banker's offer and continuing the game to the next round. If the contestant rejects all nine of the banker's offers, the contestant opens his briefcase and wins that dollar amount.

Two interesting problems arise from the way *Deal or No Deal* is setup. The first problem pertains to the decision making process of the contestant and involves when the player is making a good or bad decision (Kestenbaum, 2006). A good decision can occur when the player accepts a banker's offer that is of greater value than the player's briefcase, or when the player rejects a banker's offer that is of lower value than the player's briefcase. A bad decision simply occurs when a player does the opposite of making a good decision. The second problem involves the banker and issues such as how the banker determines his offers and when the banker gives the highest or lowest offer (Post, Baltussen and Van den Assem, 2006). The problem that we will be focusing on is how exactly the banker determines his offer for each of the nine rounds in *Deal or No Deal*. Very little is known about how the banker determines his offers, except that the offer appears to be determined using the dollar values that are still remaining in the game (Post, Baltussen and Van den Assem, 2006). In Round 9 for instance, if the remaining two dollar amounts in the game are \$5.00 and \$10.00 the banker's offer would be in the \$5.00 - \$10.00 range. It would be unrealistic for the banker to offer a value below \$5.00 or greater than \$10.00.

In this paper, we explore regression based statistical models for analyzing the *Deal or No Deal* data, in an attempt to determine how the banker determines each of his offers. We will fit these models by using data sets collected from 50 games of the online version and 50 games from the television version of Deal or No Deal. The banker's offers after every round in the online version were rejected to learn how the banker determines his offers for all nine rounds. This prevents an individual human induced bias from playing any role in decision making. In addition, a randomly selected briefcase was chosen as the contestants briefcase in each of the 50 online games used for model fitting. These precautions could not be taken in the television game show as contestants were able to choose any briefcase and accept a banker's offer at any time.

The rest of the paper is organized as follows. Section 2 will focus on important summary statistics of both the online and television versions of *Deal or No Deal*. In Section 3, we develop three regression based models to predict the banker's offers. Each of these three models is composed of nine separate models that correspond to the nine rounds in *Deal or No Deal*. The results from model fitting and their performance on predicting banker's offers for new data points are summarized in Section 4. Finally, Section 5 presents the concluding remarks and future work.

2. **Summary of the Game.** *Deal or No Deal* revolves around risk taking and decision making process by the contestant in each of the nine rounds. At the end of each round, when the banker makes an offer to the contestant, the player is left with a risk/reward decision to make. This decision is whether to accept or reject the banker's offer in an attempt to maximize his winnings. Another important theme in the game revolves around the banker who determines what the dollar amount of each offer is. The question we will be trying to answer is how the banker determines these offers. There are many smaller questions about the banker's offers that can be answered as an aid to our original question. For instance, (1) Does the banker

ever give a fair offer? We call the banker's offer to be fair if the offer made after i-th round is the average of the dollar values in the remaining briefcases in the game. (2) When will the banker give his most generous offer? (3) How do offers compare among the nine rounds? and (4) Is there an obvious trend in the banker's offers as the game progresses? To answer a few of these questions, we will look at the summary statistics of the data collected from the online game and the television show.

To begin with, we introduce some notation. After the i-th round ($1 \leq i \leq 9$), let Y_i denote the banker's offer, n_i denote the number of briefcases still in the game, and X_{ij}, $j = 1, ..., n_i$ be the dollar amount in the n_i briefcases. In each round a fixed number of briefcases are opened and thus removed from the game (recall 6, 5, 4, 3, 2, 1, 1, 1, 1 briefcases are opened in the nine respective rounds). Subtracting these nine values from the initial 26 briefcases generates our n_i, i.e., $n_1 = 20, n_2 = 15, n_3 = 11, n_4 = 8, n_5 = 6, n_6 = 5, n_7 = 4, n_8 = 3$ and $n_9 = 2$. These n_i are important because we wish to study the dependence of the remaining cases in the game on the banker's offer, that is, the relationship between the first banker's offer and $n_1 = 20$ predictor variables (corresponding to the 20 cases left), the second banker's offer and $n_2 = 15$ variables, and so on. Based on this notation, the banker's offer after Round i will be called fair if

$$Y_i = \frac{1}{n_i} \sum_{j=1}^{n_i} X_{ij}. \tag{1}$$

From here onwards, let $Y_{i,f}$ denote the fair (expected ideal) offer (1) after the i-th round. One of the important questions we are trying to answer is when does the banker give a fair offer. The verification of the fairness of offers requires very minimal computation and can easily be calculated using (1) during the game. The other questions are somewhat complex and require a more thorough investigation. In an attempt to answer these questions, we now look at the summary statistics of the data collected from the online and television versions of the game show.

We collected the data on the nine banker's offers and the dollar values in the 26 briefcases in the order they were opened for 50 games of the online version and 50 games of the television version (see Tables 4 and 6 for examples of how the data was collected for both versions). We will begin by first concentrating on the online version of the game. Figure 1(a) presents the round-wise summary of the banker's offers in all nine rounds for all 50 games. We can see from Figure 1(a) that the mean and the median of the banker's offers are very similar for the first five rounds of the game, while the distribution of the offers vary slightly as the variance (or spread) of the offers increases as the game progresses. For each of the last four rounds the mean is consistently greater than the median. This is consistent with the positive skewness of the distribution of these offers.

For each of the 50 games, we compute fair offers for the nine rounds using (1) and compare them with the actual offers. Figure 1(b) displays the distribution of $100p_i$, where $p_i = Y_i/Y_{i,f}$ is the proportion of the fair offer $Y_{i,f}$ (as in (1)) made to the contestant for selling his briefcase. From Figure 1(b), it is clear that the banker never gives a fair offer in the online game. The banker tends to give the lowest percentage of the fair offer in the first five rounds and an increasingly higher percentage of the fair offer in the later rounds, but he never gives an offer greater than or equal to the fair offer. In summary, although never fair, the generosity of the banker in terms of making an offer to the contestant increases as the game progresses.

The television game show data is presented in Figures 2(a) and 2(b). From Figure 2(a) we see that for Rounds 1 and 2 the mean and the median of the banker's offers are very similar, but for each of the last seven rounds the mean is always greater than the median. We can also see that the variance of the offers increases with the rounds, and that the positive skewness of the distribution of the offers is again present as in the online data. Figure 2(b) also shows that the proportion of fair offer has the similar increasing trend as compared to the online data. Interestingly, the banker may even give a fair or more than fair offer in the last three rounds of the television version of the game, which differs from the online version.

We now generate Figure 3 to show the similarities and differences between Figures 1(b) and 2(b). Figure 3 shows the medians of the distribution of $100p_i$, where $p_i = Y_i/Y_{i,f}$, for each of the nine rounds in both the online and television versions of Deal or No Deal. A major difference to note is that the medians for the television version is constantly increasing in a linear pattern, while the medians of the distribution of $100p_i$ for the online version is almost constant for Rounds 1 - 5 before increasing linearly for the remaining rounds. We also note that the medians of the distribution of $100p_i$ for the television version is mostly greater than the online version.

Although the distribution of Y_i, the banker's offer after the i-th round, changes with rounds (see Figures 1(a) and 2(a)), the actual offer Y_i appears to be closely related to the fair (expected ideal) offer $Y_{i,f}$ (notice the symmetry in the boxplots in Figures 1(b) and 2(b)). This motivates further investigation in establishing a more formal relationship between the banker's actual offer and the expected fair offer.

3. **Statistical Models.**

In this chapter, we develop two classes of statistical surrogates for modeling the banker's offers. The first model formalizes the relationship between the actual banker's offer and the (expected) fair offer (1) for each of the nine rounds as discussed in Section 2. The second class of surrogates consists of two statistical models that establish relationships between the banker's offer in the i-th round and the dollar values remaining in the game. To generate these models we used the data collected from the 50 online and television versions of the game show Deal or No Deal. Using the online game we were able to collect all 50 banker's offers for each of the nine rounds, but in the television version the contestants could make a deal with the banker after any round, and thus we have a different number of banker's offers for each round. Let M_i denote the number of data points collected after Round i (i.e., the number of games continued up to this round) in the television version game. For the data collected, $M_1 = 50, M_2 = 49, M_3 = 45, M_4 = 41, M_5 = 37, M_6 = 35, M_7 = 33, M_8 = 25$ and $M_9 = 17$. Note that each of the three models developed here is composed of nine separate models that correspond to the nine rounds of *Deal or No Deal*.

3.1. **Proportion based model.**

Based on the data collected from the online (NBC) version and the American television version of the game show *Deal or No Deal* (see Figures 1(b) and 2(b)) it is clear that the banker's actual offer, Y_i, given after the i-th round has strong association with the expected ideal offer $Y_{i,f}$ as in (1), however, the dependence appears to vary with the rounds. Thus, we propose

$$Y_i = p_i \cdot Y_{i,f} + \epsilon_i, \text{ for } i = 1, ..., 9, \tag{2}$$

where p_i denotes the unknown proportion, $\epsilon_i \sim N(0, \sigma_i^2)$ represents the replication error (or small random noise). Note that $p_i \in (0, 1)$ implies that the banker is offering less than the expected fair offer, and $p_i \geq 1$ implies a generous (more than fair) offer to the contestant. In this model, we assume that the the banker's offer in the i-th round can be completely characterized by $X_{i,j}, j = 1, ..., n_i$ through $Y_{i,f}$, and the information on individual X_{ij}'s, or the banker's offers from previous rounds is not necessary. Consequently, for every round, there are only two parameters σ_i^2 and p_i that have to be estimated. We use the least squares regression approach for estimating the parameters.

From here onwards, let $Y_{i,p} = \hat{p}_i \cdot Y_{i,f}$ denote the estimated offer using model (2). The model fits obtained using the data sets collected from both the online and the television version of the game are shown in Table 1.

Table 1. Parameter estimates and p-values of model (2) fitted using least square approach

Round	Online version			Television version		
	$100\hat{p}_i$	$95\%\, CI(100\hat{p}_i)$	p-value	$100\hat{p}_i$	$95\%\, CI(100\hat{p}_i)$	p-value
1	27.16	(20.00, 33.77)	7.16e-15	17.01	(6.05, 38.92)	1.76e-13
2	26.74	(20.00, 33.78)	<2.2e-16	27.29	(11.61, 55.70)	4.32e-14
3	25.50	(19.82, 34.00)	<2.2e-16	40.24	(22.74, 62.00)	2.16e-16
4	26.98	(20.00, 33.78)	<2.2e-16	50.88	(26.00, 78.92)	<2.2e-16
5	27.50	(21.00, 33.00)	<2.2e-16	62.16	(36.30, 91.19)	<2.2e-16
6	36.20	(30.22, 43.00)	<2.2e-16	73.52	(47.82, 97.68)	<2.2e-16
7	48.25	(40.24, 54.00)	<2.2e-16	82.83	(40.44, 109.52)	<2.2e-16
8	57.11	(50.23, 64.00)	<2.2e-16	92.77	(47.09, 114.75)	<2.2e-16
9	66.91	(60.00, 74.17)	<2.2e-16	95.64	(48.85, 108.91)	<2.2e-16

From Table 1, we can see that there are many differences between the model fits for the two types of data (online and television version). To begin with the 95% confidence intervals for p_i's for the television version are much wider compared to that of the online version. The $\hat{p}_i$ for the television version is generally much larger (except for Rounds 1 and 2), which means that the banker generally gives a larger and more generous offer in the television version. We can also see that the $\hat{p}_i$ do not change much during Rounds 1 to 5 in the online version but change significantly in the television version. One important difference is that the banker in the television version will sometimes give offers equal to or greater than the fair offer (expected value of the remaining cases). The upper limit of the banker's offer in the online version was approximately 74% of the fair value (Table 1, Round 9), while in the television version the upper limit of the banker's offer was approximately 115% (Table 1, Round 8). That is, on average the player should never accept a banker's offer in the online version, but in the television version the banker may make an offer in the last three rounds that will exceed the expected value of the remaining cases, and the player should accept the bankers offer. The most obvious similarity between the two types of data is that the percentage, p_i, of fair offer generally increases as the game progresses, and model (2) is a good fit to the data. Table 1 also presents the p-values from our model fits of each of the nine rounds using model (2). A poor fit would have a large p-value (> 0.05), whereas a good fit would have a small p-value (<0.05). In general, we see that the p-values are quite small ($<< 0.05$) which suggests that we have good model fits for all nine rounds.

We now use pictorial representation of the data to visualize the goodness of fit for model (2). Figures 4 and 5 are generated by comparing the fair offer, $Y_{i,f}$, against the actual banker's offer, Y_i, for all nine rounds of *Deal or No Deal*. Note that a good model fit would correspond to linear alignment of points with slope p_i. Figure 4 shows that we have good model fits for the latter rounds. The fan shaped plots in early rounds occur because when the fair offer, $Y_{i,f}$, is large, there can be a greater fluctuation in Y_i. Whereas a small fair offer does not allow for large variation in the actual offer. Figure 5 also presents similar evidence in that there is good model fits for Rounds 5-9 using the television data for model (2).

Although model (2) leads to good prediction for unsampled (or new) data points (see Section 4 for examples), the assumption that "the banker's offer after the i-th round can be completely characterized by $\{X_{i,j}, j = 1, ..., n_i\}$ through $Y_{i,f}$" is very strict and can be relaxed to construct more flexible statistical models.

3.2. **Linear regression model.** In this section we develop a statistical model that establishes a relationship between the banker's offer after the i-th round and the dollar values of the remaining briefcases in the game. We begin with the most obvious choice by modeling the banker's offer using linear regression. That is, the model for the banker's offer, Y_i, is

$$Y_i = \beta_0 + \sum_{j=1}^{n_i} \beta_j X_{i,j} + \epsilon_i, \tag{3}$$

where $X_{i,j}$ correspond to the j-th briefcases selected after the i-th round, and $\epsilon_i \sim N(0, \sigma_i^2)$ is the replication error. For instance, after Round 8 of the two examples (Tables 4 and 6) in Section 4 the $\{X_{i,j}, j = 1, ..., n_i\}$ are $\{X_{8,1} = \$400, X_{8,2} = \$50,000, X_{8,3} = \$100\}$ and $\{X_{8,1} = \$75, X_{8,2} = \$5,000, X_{8,3} = \$300,000\}$. Note that if β_j, $j = 1, ..., n_i$, are all equal and $\beta_0 = 0$ then model (3) becomes model (2).

When we fit this model to the data sets from different rounds and sources (online version and the television version) the R^2_{adj} values for each of the nine rounds are extremely close to 1, and therefore all of the coefficients (β_0, β_1,..., β_{n_i}) are statistically significant (i.e., non-zero). This occurs because the dollar values are assigned to random briefcases and $\{X_{i,j}, j = 1, ..., n_i\}$ are the dollar values in the n_i briefcases in the order they were selected and opened. Since $X_{i,j}$ are associated with the dollar value in the j-th briefcase and not the j-th dollar value in the remaining briefcases, the predictor variables have no inherent meaning (ordering) attached to them.

The order in which the briefcases are opened should have no influence on the banker's offer because the offer is based on only the dollar values remaining in the game. Thus, we need to adjust our predictor variables in order to get a better understanding behind the banker's offer. This motivates the transformation of $\{X_{i,j}, j = 1, 2, ..., n_i\}$ to $\{X_{i(j)}, j = 1, 2, ..., n_i\}$, where $X_{i(j)}$ denotes the j^{th} smallest dollar value in $\{X_{i,j},\ j = 1, 2, ..., n_i\}$. The revised linear regression model is given by,

$$Y_i = \beta_0 + \sum_{j=1}^{n_i} \beta_j X_{i,(j)} + \epsilon_i, \tag{4}$$

where $X_{i,(j)}$ is the j-th lowest dollar value among the briefcases remaining in the game after in the i-th round, Y_i is the banker's offer after the i-th round, n_i is the number of briefcases remaining in the game, and $\epsilon_i \sim N(0, \sigma_i^2)$ is the replication error.

Ordering the variables in this way enables us to interpret the results and comment on which variables are most/least important. The predictors $X_{i,(j)}$ for Round 8 in the two listed games (Tables 4 and 6) will now become $\{X_{8,(1)} = \$100,\ X_{8,(2)} = \$400,\ X_{8,(3)} = \$50,000\}$ and $\{X_{8,(1)} = \$75,\ X_{8,(2)} = \$5,000,\ X_{8,(3)} = \$300,000\}$. Although model (4) solves the problem of the variables being non-interpretable, there are several diagnostics that still need to be preformed to determine if the model assumptions are violated, and therefore needs further investigation.

There are a few key assumptions that a linear regression model should satisfy in order to be deemed acceptable (Kutner, Nachtsheim, Neter and Li, 2005, Chapter 3). These assumptions include linearity, constant variance and normality. It turns out that model (4) violates the constant variance assumption, and the error variance is an increasing function of $\hat{Y}_i^*$. This problem occurs because the increments between the consecutive dollar values in the 26 briefcases are non linear (in fact they appear exponential) when the values are arranged from smallest ($0.01) to largest ($1,000,000), see Figure 6 for all of the 25 increments on the log-scale. Since this model violates the constant variance assumption, it is also not a desired model to predict the banker's offers.

Taking the logarithm of the predictor variables $X_{i(j)}$ (j-th smallest dollar value remaining in the game) and banker's offers, Y_i, eliminates the non-linearity in the relationship, and satisfies all the model assumptions. The new adjusted model is

$$Y_i^* = \beta_0 + \sum_{j=1}^{n_i} \beta_j X_{i,(j)}^* + \epsilon_i, \tag{5}$$

where $Y_i^* = \log(Y_i)$ and $X_{i,(j)}^* = \log(X_{i,(j)})$ for $j = 1, ..., n_i,\ i = 1, ..., 9$, and $\epsilon_i \sim N(0, \sigma_i^2)$ accounts for the replication error. We can now look at the fit of model (5) using the data collected from both the online and television versions of Deal or No Deal. Table 2 presents the adjusted R^2 values and p-values of the overall model fits for each of the nine rounds. A poor fit would have an adjusted R^2 value close to zero, whereas a perfect fit would have an adjusted R^2 value of one.

TABLE 2. Adjusted R^2 values and p-values from model (5) fit

Round	Online version		Television version	
	R^2_{adj}	P-value	R^2_{adj}	P-value
1	0.5779	5.92e-10	0.4295	5.66e-06
2	0.7373	8.54e-15	0.4493	3.76e-08
3	0.8989	<2.2e-16	0.8068	6.20e-14
4	0.9330	<2.2e-16	0.8704	2.69e-16
5	0.9785	<2.2e-16	0.8933	<2.2e-16
6	0.9844	<2.2e-16	0.9357	<2.2e-16
7	0.9916	<2.2e-16	0.9759	<2.2e-16
8	0.9960	<2.2e-16	0.9732	<2.2e-16
9	0.9980	<2.2e-16	0.9844	<2.2e-16

From Table 2 we see that both the online and television versions have a similar pattern in that the fit gradually gets better as the game progresses. In other words, the banker's offers have more variation in the earlier rounds compared to the later rounds. We can also see that model (5) for the online version has a slightly better fit, larger adjusted R^2 values for every round, compared to the television version model. Given the goodness of fit, this model should allow for good predictions of unsampled data points in later rounds and reasonable predictions in the earlier rounds.

Although intuitive, model (5) is relatively complicated because it requires several transformations (i.e., ordering, logarithms), and varying number of predictors for different rounds, thus we now attempt to propose a simpler model.

3.3. **Indicator based model.** In this section, we use 26 predictor variables (associated with the dollar values in the 26 briefcases) for modeling the banker's offer after each round. To begin with, define indicator variables,

$$W_{i,j} = \begin{cases} 1 & \text{if the } j\text{-th smallest dollar value (out of 26) is still in the game after Round } i \\ 0 & \text{otherwise} \end{cases}$$

for $j = 1, ..., 26$, and $i = 1, ..., 9$. The proposed model is given by,

$$Y_i^* = \beta_0 + \sum_{j=1}^{26} \beta_j W_{i,j} + \epsilon_i, \tag{6}$$

where $W_{i,j}$ correspond to the 26 dollar amounts (in increasing order from \$0.01 to \$1,000,000) and $\epsilon_i \sim N(0, \sigma_i^2)$ measures the replication error. For instance, $W_{i,1} = 1$ represents that the smallest dollar value, \$0.01, is still in the game and $W_{i,26} = 0$ means the largest dollar value, \$1,000,000, is no longer in the game after the i-th round. In this model, the predictor variables have slightly different interpretations and are directly associated with the dollar values instead of the briefcases left in the game after the i-th round, which was the case in models (2) and (5).

The purpose of using these indicators is to determine which of the dollar values are most important in the game for influencing the banker's offers. Using model (5), we were able to determine if the highest or lowest dollar values among the ones still in the game play a crucial role in predicting the banker's offer, however, identifying which specific dollar amounts were the most important was impossible. Model (6) will enable us to determine which dollar amounts are most important. Table 3 presents the adjusted R^2 and p-values of the model fit for every round using both data sets collected from the online version and television version of the game show.

Examining Table 3 and comparing it with Table 2, we see many similarities. For instance, the goodness of fit for the models gradually increase as the game progresses. The R^2_{adj} values for the television version and online version, again show the similar pattern, that is, the online data appears to be a better fit for model (6) compared to the television data. The only noticeable difference between Table 3 and Table 2, is that the R^2_{adj} values and p-values are smaller using model (6) leading us to believe that predictions on unsampled data points may be further away from the actual banker's offers.

4. **Examples.** To illustrate the three models developed in Section 3, we now present a few examples based on new data points collected from the online and television versions of the game show, *Deal or No Deal*. To compare the performance of these three models, we use graphical techniques and numerical summaries

TABLE 3. Adjusted R^2 values and p-values from model (6) fit

Round	Online version		Television version	
	R^2_{adj}	P-value	R^2_{adj}	P-value
1	0.5805	6.54e-11	0.4976	7.56e-09
2	0.6974	9.72e-13	0.6158	4.87e-12
3	0.8316	1.33e-15	0.7536	8.45e-14
4	0.9089	<2.2e-16	0.8582	1.63e-16
5	0.8971	2.26e-16	0.8665	1.72e-16
6	0.9730	<2.2e-16	0.8773	1.83e-16
7	0.9719	<2.2e-16	0.8815	1.86e-16
8	0.9872	<2.2e-16	0.8849	1.95e-16
9	0.9928	<2.2e-16	0.9374	<2.2e-16

in the form of average differences between predicted and actual banker's offers from the online and television versions of the game. First we discuss the predictions performance of the models for the online data, then new data points from the American television show are used to measure the prediction efficiency of the three models.

Online Data. We initially collected 50 data points by playing the online version of *Deal or No Deal* hosted on the NBC website to fit the three working models developed in Section 3. We now collect three new data points for comparing the prediction performance of the three models. Table 4, presents round-by-round break down of the dollar values in the briefcases as they were revealed and the banker's offers for the first new data point. Note that Table 4 has a column titled "Extras" that correspond to the player's briefcase ($100) that was selected to start the game and one additional briefcase ($50,000) that was never opened. We now use this information to predict the banker's offers using models (2), (5), and (6) that were fitted using the 50 data points collected earlier. The last row of each column in Table 4 correspond to the actual banker's offers.

The left panel of Figure 7(a) compares the predicted offers with the actual banker's offers for the nine rounds. Figure 7(a) presents the actual and predicted banker's offers using all three models, for the online game. We see that all three models predict the actual banker's offers reasonably well as none of the predicted offers appear too far from the actual banker's offer. One noticeable characteristic is that for different rounds different models perform better at predicting the actual banker's offer, and there doesn't appear to be any immediate pattern. A closer look at the left panel of Figure 7(a) shows that model (5) is furthest from the actual offers in Rounds 1, 5, 6 and 7, predicted offers from model (6) is furthest in Rounds 2 and 4, and finally all three models are equally good at predicting the banker's offers in the remaining rounds. It may appear from Figure 7(a) that model (2), the proportion based model, most accurately predicts the banker's offer in all nine rounds and therefore should be considered the best model, but this is not necessarily true when we examine the prediction performance of these models for the second and third new data points from the online game. The left panels of Figures 7(b) and 7(c) present the comparison of the actual offers with the predicted offers obtained from the three models in all nine rounds.

Solely looking at the left panels in Figures 7(a), 7(b) and 7(c) is not enough to compare the performance of these three models at predicting the actual banker's

TABLE 4. The dollar values opened in each round along with the corresponding banker's offers for the first new point of the online version of *Deal or No Deal*

Round 1	Round 2	Round 3	Round 4	Round 5
5 10,000 25 5,000 100,000 10	50 1,000 500,000 75 750,000	750 500 200,000 0.01	300 75,000 25,000	300,000 400,000
44,596	27,363	45,433	50,333	52,535
Round 6	Round 7	Round 8	Round 9	Extras
200	1	1,000,000	400	50,000 100
77,737	136,656	8,921	17,786	

offers. Thus, we also use the margin of error for prediction intervals of the three models and the average distance (prediction error) from the actual banker's offers.

The smaller the margin of error the more accurate the model will be at predicting the banker's offer. The right panel of Figure 7(a) corresponds to the margin of error for the prediction of the banker's offers for the first new data point from the online game, shown in the left panel of Figure 7(a). Similarly, the right panels of Figures 7(b) and 7(c) display the corresponding margin of errors for the prediction intervals of the other two new data points. The right panel of Figure 7(a) shows that no model has a consistent low or high margin of error, which again agrees with the no obvious winner model. The right panels of Figures 7(b) and 7(c) also support this conclusion.

Finally, we look at the average difference in the predictions via Root Mean Square Prediction Error (RMSPE) for each of our models. The RMSPE is given by

$$\sqrt{\frac{1}{9}\sum_{i=1}^{9}(Y_i - \hat{Y}_i)^2}, \tag{7}$$

where Y_i is the actual banker's offer after the i-th round and $\hat{Y}_i$ is the predicted banker's offer after the i-th round. Table 5 show the RMSPE values for each of the three new data points obtained from the online games and three different models. From Table 5 we can see that the proportion based model (2) has the smallest RMSPE values, and the linear regression (log ordered) model (5) has the largest RMSPE values.

In summary, the results show that all three of our models are capable of predicting banker's offers reasonably close to the actual banker's offers for each of the nine rounds. Though the prediction plots and the margin of error plots in Figure 7 do

Table 5. RMSPE values in dollars from each model's prediction for the online data

New Data	Percentage model	Log Ordered model	Binary model
Point 1	5,605.06	7,940.75	5,778.31
Point 2	13,908.43	23,734.64	18,633.90
Point 3	7,535.10	17,776.39	8,768.63

not show a clear winner, Table 5 suggests that, on average, models (2) and (6) lead to closest predictions for the banker's offers.

Television Data. Similar as in the online version of the game, we first collected 50 data points from the television version of the game for model fitting, and then we collected three new data points for assessing the performance of our three models in predicting the banker's offers. Table 6 presents the first new data point from the American version of the television game show *Deal or No Deal*. The dollar values in column "Extras" correspond to the player's briefcase ($300,000) and one additional briefcase ($5,000) that was never opened. Apart from the order in which the briefcases are opened (hence the ordering of the dollar values), the only noticeable difference from the online version, is that the banker's offers in Table 6 appear to be "nice" numbers as they are proposed by a real person (the banker), and they are perhaps rounded, whereas the offers in the online version are generated automatically using some computer program.

Table 6. The dollar values opened in each round along with the corresponding banker's offers for the first new point of the television version of *Deal or No Deal*

Round 1	Round 2	Round 3	Round 4	Round 5
10,000 0.01 1 500 1,000 200	25 400,000 10 5 400	50,000 1,000,000 300 500,000	750,000 750 75,000	100,000 100
29,000	66,000	43,000	37,000	50,000
Round 6	Round 7	Round 8	Round 9	Extras
50	25,000	200,000	75	5,000 300,000
81,000	124,000	63,000	140,000	

The left panel of Figure 8(a) presents the actual and predicted banker's offers for all nine rounds using the data from Table 6. Figure 8(a) shows that the prediction from model (6) is furthest from the actual banker's offers in Rounds 2, 3, 4 and 8; prediction from model (5) is furthest in Rounds 7, and all models have very close predictions to one another in the remaining rounds as in the online version. It may appear that model (2) is the best at predicting the actual banker's offers, however, the two additional new points show that this is not always the case (see Figures 8(b) and 8(c)).

Similar to the online version, we will look at the margin of error and RMSPE for the banker's offers as a means of distinguishing how well the three models are able to predict the actual banker's offers. The margin of error plots here (the right panels of Figures 8(a), 8(b) and 8(c)) show distinct characteristics as compared to right panels of Figures 7(a), 7(a) and 7(c). Focusing on the right panel of Figure 8(a) we notice that the margin of error lines appear to be fairly separated and that model (6) generally has the largest margin of error and model (2) has the lowest margin of error. It turns out that the trend prevails in the other two new data points (see the right panels of Figures 8(b) and 8(c)).

Table 7 summarizes the RMSPE values for the banker's offers in all nine rounds for the three new games. Clearly, model (2) has the smallest RMSPE values and model (6) has the largest RMSPE values. Comparing Tables 7 and 5 we notice that model (2) has the smallest RMSPE values in both the television and online versions of the game for all three new data points. In general, the RMSPE values appear much larger in Table 7 (television version) than in Table 5 (online version). That is, the predictions for the television data are worse compared to the online data.

Table 7. RMSPE values in dollars from each model's prediction for the television data

New Data	Percentage model	Log Ordered model	Binary model
Point 1	13,420.55	28,309.79	34,203.64
Point 2	3,082.75	9,041.76	10,750.19
Point 3	26,574.84	27,239.68	71,986.11

Comparing the left panel of Figures 8(a), 8(b) and 8(c) to the left panel of Figures 7(a), 7(b) and 7(c), one can see some obvious differences. A quick glance of Figures 8(a), 8(b) and 8(c), reveals that the predictions for the three models tend to differ from each other in most rounds. This is different from the online data results (Figures 7(a), 7(b) and 7(c)) because in many of the rounds, the predictions from all three models are almost indistinguishable. Moreover, we see that the predictions for the banker's offers in the television version (left panels of Figures 8(a), 8(b) and 8(c)) appear to be further away from the actual banker's offer compared to that of the predictions for the banker's offers in the online version (left panels of Figures 7(a), 7(b) and 7(c)). In summary, we see that all three models have reasonable predictions of the banker's offers but model (2) has the smallest margin of error and RMSPE values, making it the most favorable model.

5. **Discussion and Conclusion.** The game show *Deal or No Deal* presents a nice application to several mathematicians, statisticians, and economists as a natural decision making experiment. The focus of this paper was to determine if a model could be constructed to discover how the banker determines his offers. While trying to do this, data from both the online and television versions of *Deal or No Deal* were analyzed in an attempt to explain the connection between the 26 dollar amounts and the banker's offers. We developed three different regression based models to potentially predict the banker's offers.

The first model, given by (2), uses a percentage of the average dollar value still remaining in the game to predict the banker's offers. Our next model, (5), went through a few transformations and resulted in a linear regression model where the predictors are the logarithm of the ordered variables that corresponded to dollar values from lowest to highest. Both of these models only took into account the remaining variables that were still in the game at the time of the offer. Finally, model (6) was constructed using an indicator (binary) based approach and involved all 26 dollar values for each of the nine banker's offers. These three models were then compared to determine which, if any, gave the best/worst predictions of the banker's offers.

For the online version, it seemed that all three models were able to predict the banker's offer quite well and none of the models had clear advantage over the other ones. For the television version, model (2) was an obvious winner as it generally produced the closest predictions to the actual banker's offers but both models (5) and (6) also predicted the banker's offers reasonably well. It would especially be very interesting to see if there exists such a model that accurately predicts the banker's offers in *Deal or No Deal*. In this paper, we assumed that the banker's offer for Round i does not depend on the offers from the previous rounds, perhaps, relaxing this assumption may lead to a better model.

Focusing on only how the banker makes an offer resulted in many of the other problems in *Deal or No Deal* being left unanswered. Other possible problems for further investigation include, (1) At what point should a player reject or accept the banker's offer? (2) When will the banker give a player the best or the worst offer? These types of questions depend strongly on the individual player. Each individual will have a different opinion of what a good (or rather appealing) offer is (Kahneman, 1979). For example, a wealthier person may not feel that a banker's offer is ever high enough, whereas a poorer person may feel obligated to except a less than "fair" offer. The player dependency makes these types of questions very hard to answer, even using statistical models (Kachelmeier and Shehata, 1992).

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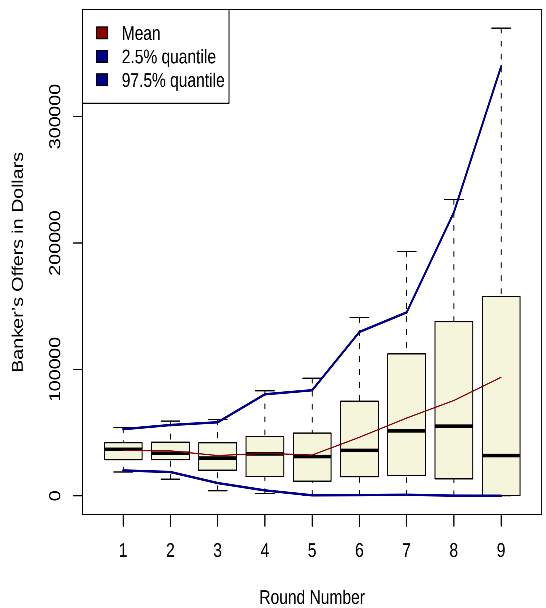
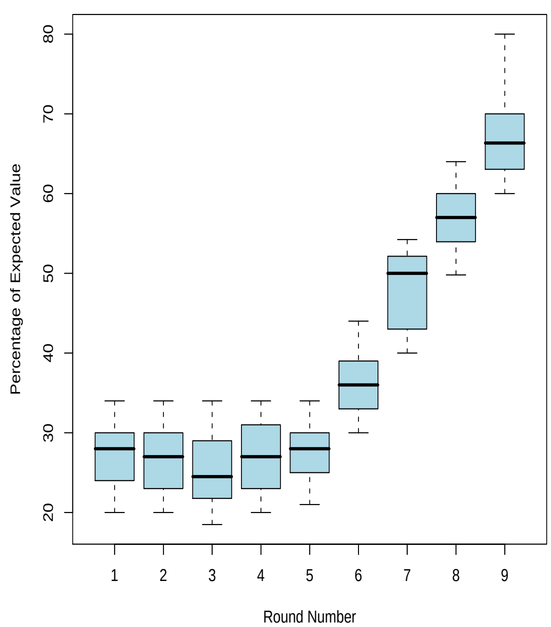
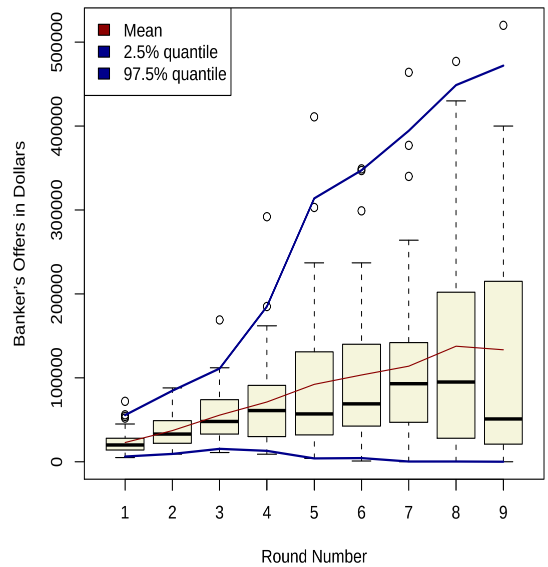


(a) Banker's offers

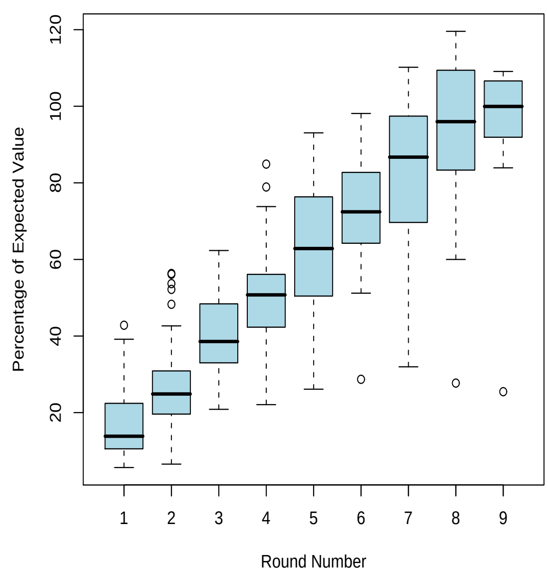


(b) Proportion of fair offers

FIGURE 1. Online data. (a) The red line represents the mean, and the area between the blue lines represents a 95% confidence interval. (b) The distribution of the $100p_i$, where $p_i = Y_i/Y_{i,f}$ is the proportion of the fair offer (2.1).



(a) Banker's offers



(b) Proportion of fair offers

FIGURE 2. Television data. (a) The red line represents the mean, and the area between the blue lines represents a 95% confidence interval. (b) The distribution of the $100p_i$, where $p_i = Y_i/Y_{i,f}$ is the proportion of the fair offer (2.1).

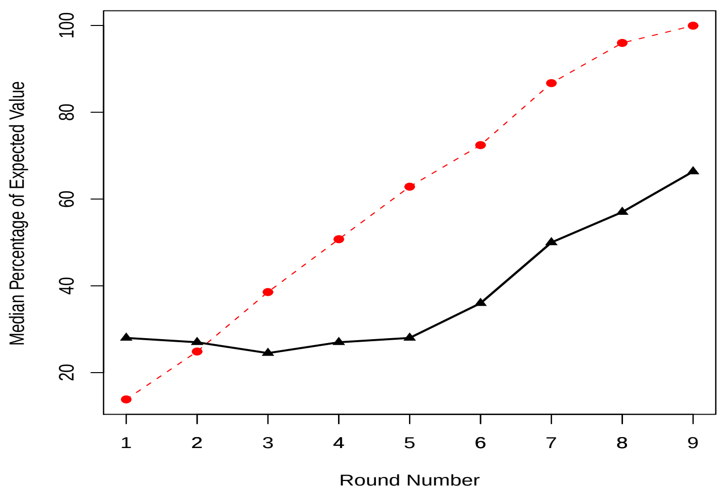


FIGURE 3. A comparison between the median percentage of the fair offers for the online version (solid black line) and the television version (dashed red line).

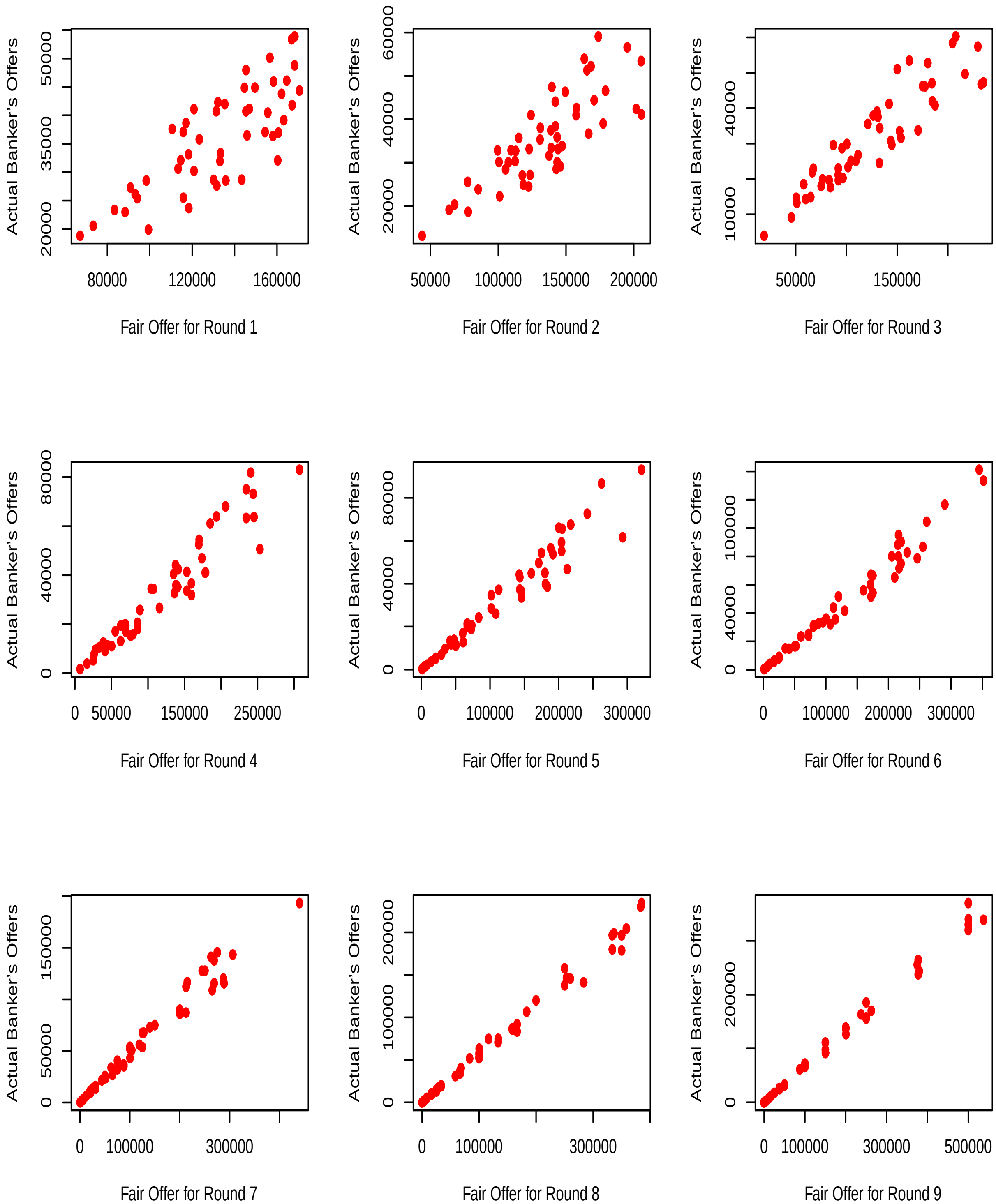


FIGURE 4. Round-by-round comparison of the Y_i vs. $Y_{i,f}$ for the 50 online games.

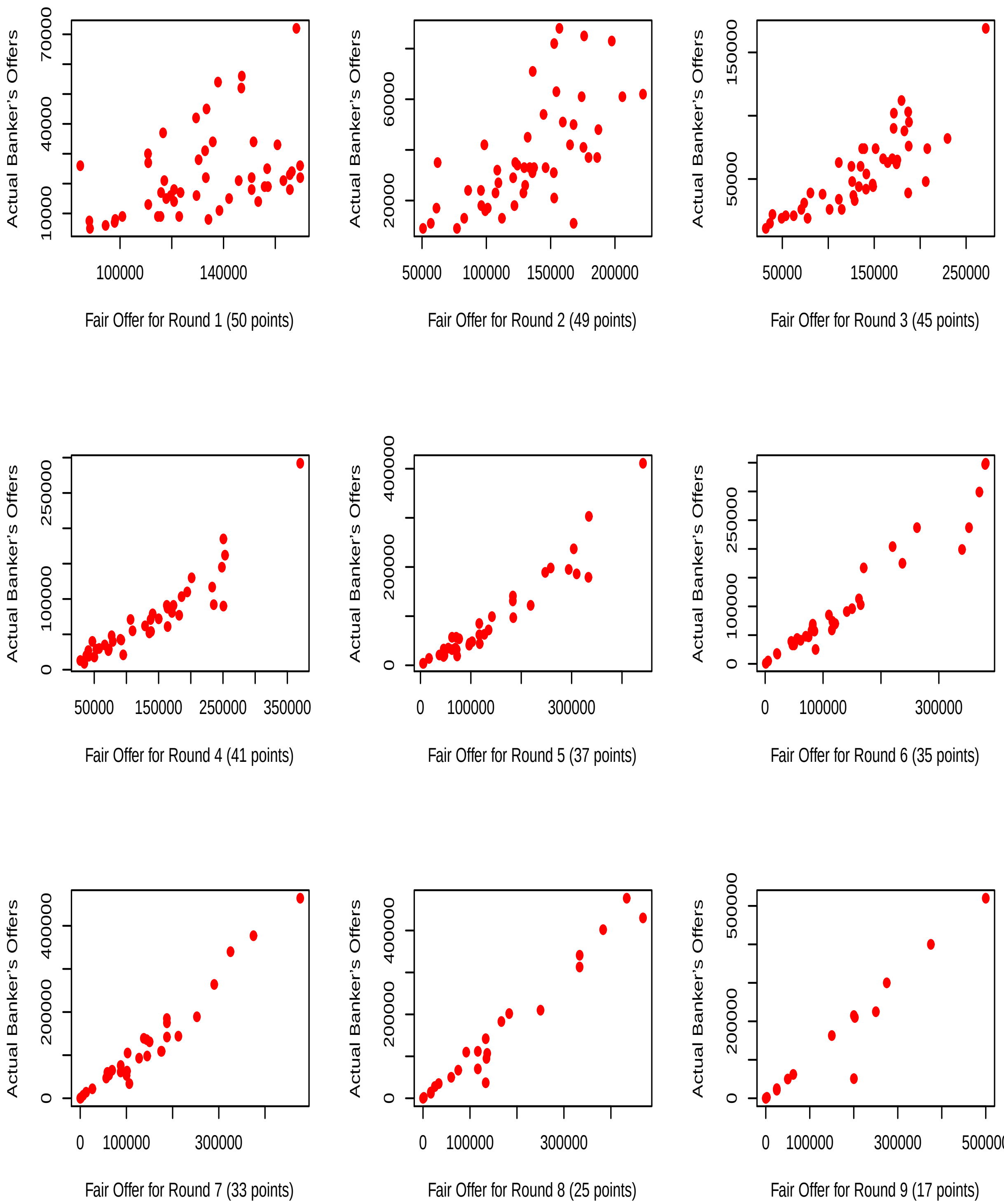


FIGURE 5. Round-by-round comparison of the Y_i vs. $Y_{i,f}$ for the 50 television games.

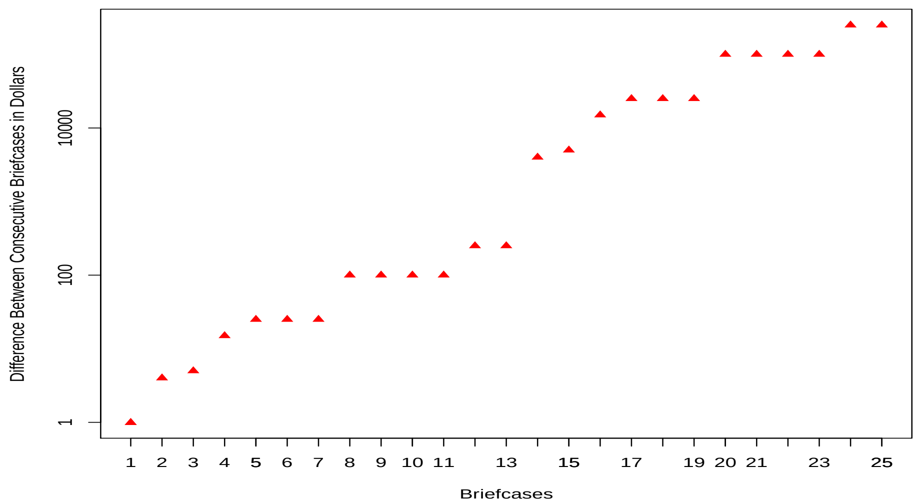
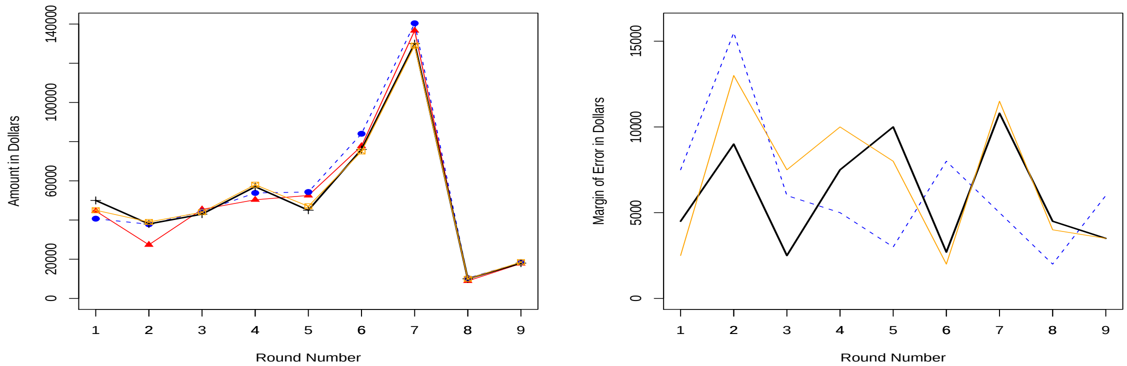
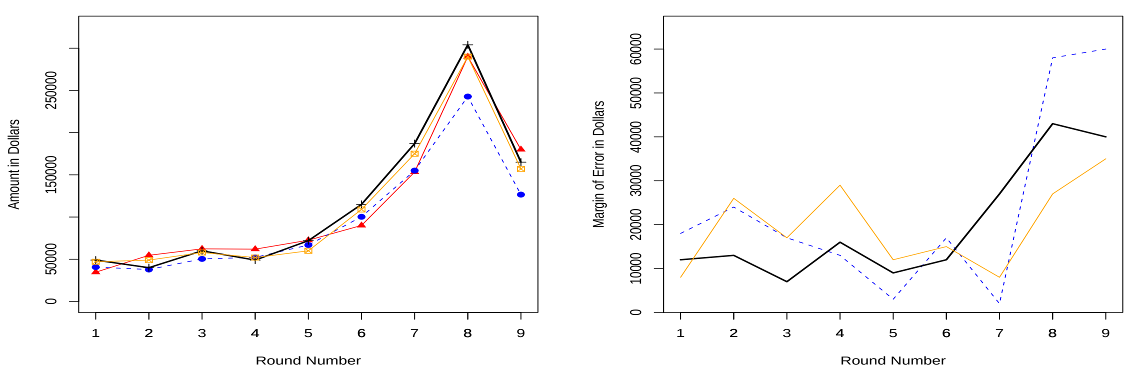


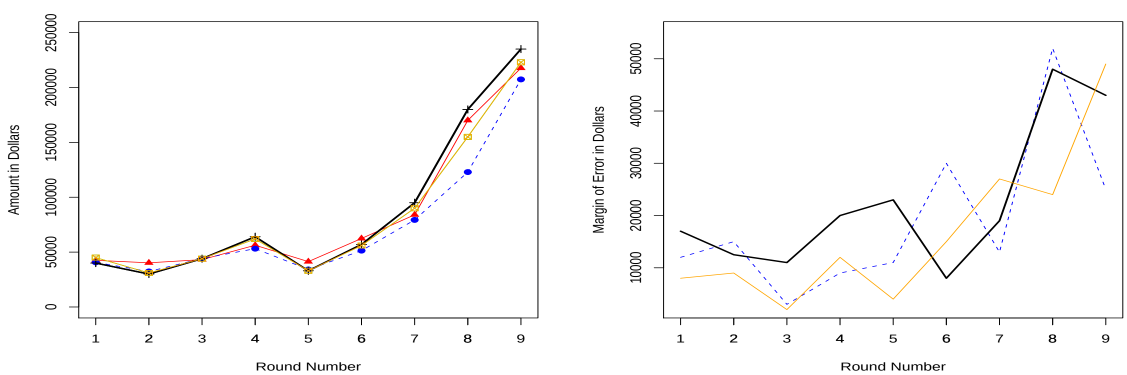
FIGURE 6. The increments between consecutive dollar values of the 26 briefcases on $\log_e$ scale.



(a) Game 1: left panel - predicted vs actual banker's offers; right panel - margin of error

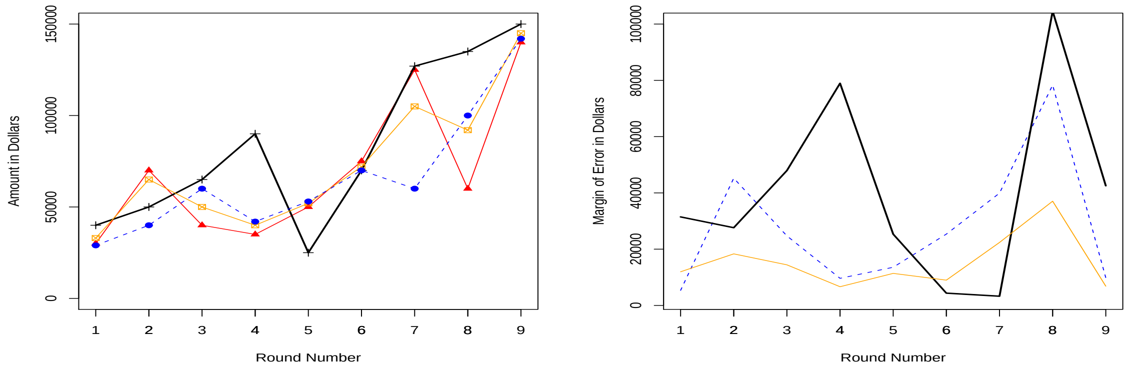


(b) Game 2: left panel - predicted vs actual banker's offers; right panel - margin of error

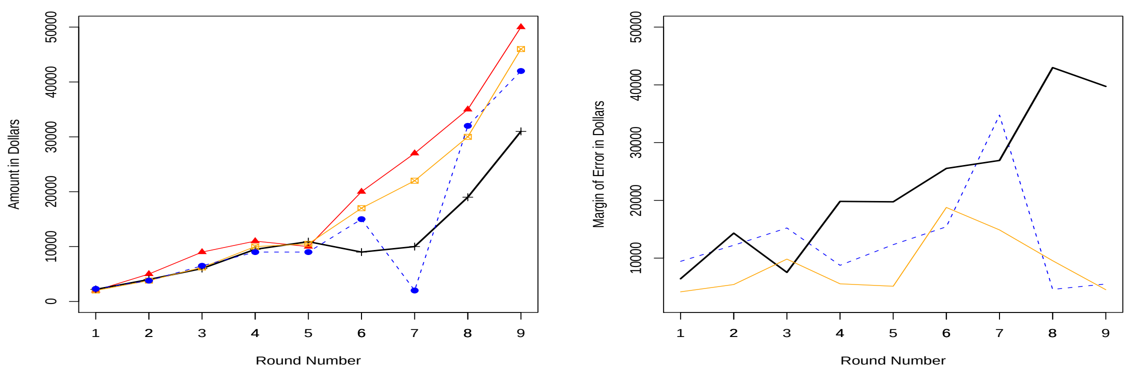


(c) Game 3: left panel - predicted vs actual banker's offers; right panel - margin of error

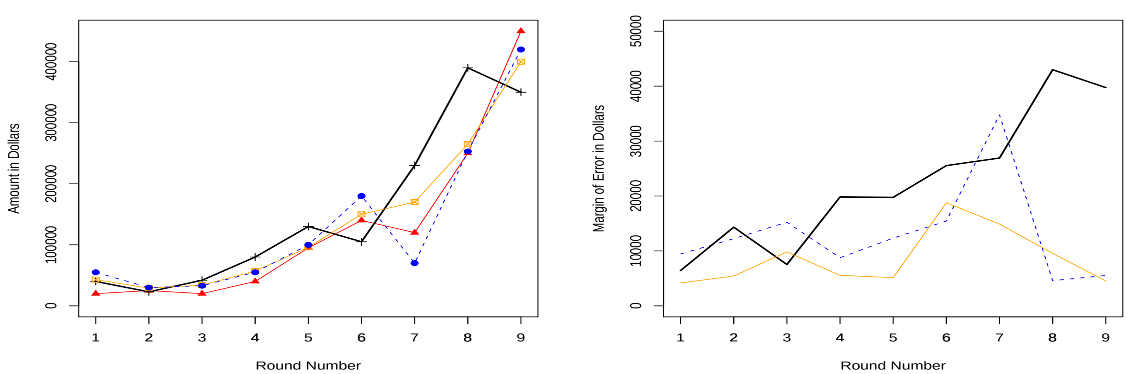
FIGURE 7. The red (triangle) represents the actual banker's offers, orange (square) represents model (2), blue (circle) represents model (5), and black (plus) represents model (6).



(a) Game 1: left panel - predicted vs actual banker's offers; right panel - margin of error



(b) Game 2: left panel - predicted vs actual banker's offers; right panel - margin of error



(c) Game 3: left panel - predicted vs actual banker's offers; right panel - margin of error

FIGURE 8. The red (triangle) represents the actual banker's offers, orange (square) represents model (2), blue (circle) represents model (5), and black (plus) represents model (6).