**Mathematics 3790H – Analysis I: Introduction to analysis**
TRENT UNIVERSITY, Winter 2012

**Solution to Assignment #3**

**1.** Give an example of a sequence $a_n$ which satisfies the condition

*For all* $\varepsilon > 0$ *there is a* $N$ *such that for all* $n \geq N$, $|a_n - a_{n+1}| < \varepsilon$.

but which does *not* converge. *[10]*

HINT: Compare the given condition to the Cauchy Convergence Criterion for sequences (§2.12 in the text). They're *almost* the same ...

SOLUTION. Let $a_n = \sum_{k=1}^{n} \frac{1}{k} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$. Then $a_n$ does not converge (this was done in class, and it is Example 2.12 in the text), but it does satisfy the given condition:

Suppose $\varepsilon > 0$ is given. Choose any integer $N \geq \frac{1}{\varepsilon}$. Then if $n \geq N$, we have

$$|a_n - a_{n+1}| = \left| \left( \sum_{k=1}^{n} \frac{1}{k} \right) - \left( \sum_{k=1}^{n+1} \frac{1}{k} \right) \right| = \left| \frac{-1}{n+1} \right| = \frac{1}{n+1} < \frac{1}{n} \leq \frac{1}{N} \leq \varepsilon \,,$$

as desired. ■