

**Mathematics 3790H – Analysis I: Introduction to analysis**  
TRENT UNIVERSITY, Winter 2012

**Assignment #7**

**Find the limit!**

*Due on Thursday, 8 March, 2012.*

1. Suppose we define a sequence  $a_n$  as follows:  $a_0 = \frac{1}{2}$  and  $a_{n+1} = \frac{1}{1+a_n}$  for  $n \geq 0$ .  
Show that this sequence converges and find its limit. [10]

NOTE: To save the empirically inclined a little effort, here are the first few elements of the sequence:

$n$	$a_n$	decimal
0	$\frac{1}{2}$	0.5
1	$\frac{1}{1+\frac{1}{2}} = \frac{2}{3}$	0.66666666666666...
2	$\frac{1}{1+\frac{2}{3}} = \frac{3}{5}$	0.625
3	$\frac{1}{1+\frac{3}{5}} = \frac{5}{8}$	0.61538461538461...
4	$\frac{1}{1+\frac{5}{8}} = \frac{8}{13}$	0.61904761904761...
5	$\frac{1}{1+\frac{8}{13}} = \frac{13}{21}$	0.61764705882352...
$\vdots$	$\vdots$	$\vdots$