

Mathematics 3790H – Analysis I: Introduction to analysis
TRENT UNIVERSITY, Winter 2012

Assignment #10
A pathological Taylor series ...
Due on Thursday, 29 March, 2012.

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by $f(x) = \begin{cases} e^{-1/x^2} & x \neq 0 \\ 0 & x = 0 \end{cases}$.

1. Show that

$$f^{(n)}(x) = p_{3n}\left(\frac{1}{x}\right)e^{-\frac{1}{x^2}}$$

for all $x \neq 0$ and all $n \geq 1$, where p_{3n} is a polynomial of degree $3n$. [4]

HINT: This is probably best done by induction on n .

2. Show that $f^{(n)}(0) = 0$ for all $n \geq 1$. [4]

3. What is the Taylor series of $f(x)$ at 0? What is its radius of convergence? [1]

4. For what x is $f(x)$ equal to its Taylor series at 0? [1]

NOTE: This function is the poster child for the fact that a Taylor series for a function need not converge to the function it came from ...