Mathematics 3790H – Analysis I: Introduction to analysis
TRENT UNIVERSITY, Fall 2010

Quizzes

Quiz #1. ~~Thursday, 23~~ ~~Tuesday, 28~~ Thursday, 30 September, 2010 *(7 minutes)*

1. Show that there is no smallest positive real number. *[5]*

Quiz #2. Thursday, 30 September, 2010 *(8 minutes)*

1. Show that the sequence $s_n = \dfrac{n-1}{n}$ has a limit. *[5]*

Quiz #3. ~~Thursday, 7~~ Tuesday, 12 October, 2010 *(10 minutes)*

1. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n3^n}$ converges or not. *[5]*

Quiz #4. ~~Thursday, 14~~ Wednesday, 20 October, 2010 *(10 minutes)*

1. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(5^n)^n}{n!e^n}$ converges or not. *[5]*

Quiz #5. Thursday, 21 October, 2010 *(10 minutes)*

1. For which values of a does the series $\displaystyle\sum_{n=0}^{\infty} \frac{1}{a^{2n}+1}$ converge? *[5]*

Quiz #6. Thursday, 4 November, 2010 *(10 minutes)*

1. Find the Taylor series at $a = 1$ of $f(x) = \ln(x)$. *[5]*

Quiz #7. Thursday, 11 November, 2010 *(15 minutes)*

1. Suppose $p(x) = a_k x^k + a_{k-1}x^{k-1} + \cdots + a_1 x + a_0$ is a polynomial of degree k. Show that the Taylor series at $a = 0$ of $p(x)$ is equal to $p(x)$. *[5]*

Quiz #8. Thursday, 18 November, 2010 *(15 minutes)*

1. Suppose $f(x) = \sum_{n=0}^{\infty} a_n x^n$ is a series with radius of convergence $R > 0$ and $[a, b] \subset (-R, R)$. Why is $f(x)$ bounded on $[a, b]$?

Quiz #8. *Alternate version. (15 minutes)*

1. Suppose $f(x) = \sum_{n=0}^{\infty} a_n x^n$ is a series with radius of convergence $R > 0$ and $f(x) = 1$ is constant on some closed interval $[-b, b] \subset (-R, R)$, where $b > 0$. Determine a_n for all $n \geq 0$.

Quiz #9. ~~Thursday, 25~~ Tuesday, 30 November, 2010 *(15 minutes)*

1. Show that $\dfrac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots = \sum_{n=0}^{\infty} (-1)^n x^{2n}$. *[2]*

2. Find the Taylor series at 0 of $\arctan(x) = \int_0^x \dfrac{1}{1+t^2}\, dt$. *[3]*

Quiz #9. *Alternate version. (15 minutes)*

1. Show that $e^{x^2} = 1 + x^2 + \dfrac{x^4}{2} + \dfrac{x^6}{6} + \cdots = \sum_{n=0}^{\infty} \dfrac{x^{2n}}{n!}$. *[2]*

2. Find the Taylor series at 0 of $f(x) = \int_0^x e^{x^2}\, dt$. *[3]*

Quiz #10. Thursday, 2 December, 2010 *(15 minutes)*

1. Recall that $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{n=0}^{\infty} x^n$ for $x \in (-1,1)$. Show that the series cannot converge uniformly to $\dfrac{1}{1-x}$ over the whole interval $(-1,1)$.

   Hint: $\dfrac{1}{1-x}$ has an asymptote at $x = 1$.

Quiz #11. Thursday, 2 December, 2010 *(15 minutes)*

1. Suppose $\sum_{n=0}^{\infty} a_n$ converges absolutely. Show that $\sum_{n=0}^{\infty} a_n \cos(nx)$ converges for all x.

Quiz #11. *Alternate version. (15 minutes)*

1. Why can't there be a sequence of differentiable functions $f_n(x)$ such that $f_n(x)$ converges uniformly to $f(x) = |x|$ on $(-1,1)$?