Mathematics 3790H – Analysis I: Introduction to analysis
TRENT UNIVERSITY, Fall 2010

Assignment #6
Due on Thursday, 9 December, 2010.

Using Gauss' Test

Before tackling this assignment, be sure to at least skim through *Gauss' Test,* an online appendix to *A Radical Approach to Real Analysis* (2nd edition), by David M. Bressoud (2006). This gives a more detailed version of Gauss' Test than our textbook. It can be found at: `http://www.macalester.edu/aratra/edition2/chapter4/chapt4d.pdf`

Suppose α, β, and γ are any real numbers not in $\mathbb{Z}^{\leq 0} = \{0, -1, -2, \dots\}$, and consider the following power series:

$$1 + \frac{\alpha \cdot \beta}{1 \cdot \gamma} x + \frac{\alpha(\alpha+1) \cdot \beta(\beta+1)}{1 \cdot 2 \cdot \gamma(\gamma+1)} x^2 + \frac{\alpha(\alpha+1)(\alpha+2) \cdot \beta(\beta+1)(\beta+2)}{1 \cdot 2 \cdot 3 \cdot \gamma(\gamma+1)(\gamma+2)} x^3 + \cdots$$
$$= 1 + \sum_{n=1}^{\infty} \frac{\alpha(\alpha+1)\dots(\alpha+n-1) \cdot \beta(\beta+1)\dots(\beta+n-1)}{n! \cdot \gamma(\gamma+1)\dots(\gamma+n-1)} x^n$$

This is what used to be called a hypergeometric series before the more general definition now in use (and used in Bressoud's book) came along.

1. Why are the constants α, β, and γ not allowed to be 0 or any negative integer in the definition above? *[1]*

2. Show that Newton's binomial series is a series of this type. *[1]*

3. Determine for which values of x this series converges absolutely, converges conditionally, and diverges, respectively. *[8]*