Mathematics 3790H – Analysis I: Introduction to analysis
TRENT UNIVERSITY, Fall 2010

Assignment #5
Alternatives?
Due: Friday, 26 November, 2010

Recall that the harmonic series, $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$, diverges, while its close relation, the alternating harmonic series, $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$, converges.

1. Determine whether the following relative of the harmonic series converges or diverges: $1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} + \frac{1}{8} - \frac{1}{9} + \cdots = \sum_{n=1}^{\infty} \frac{\beta(n)}{n}$, where $\beta : \mathbb{N}^+ \to \{-1, 1\}$ is given by $\beta(n) = \begin{cases} +1 & n \neq 0 \pmod 3 \\ -1 & n = 0 \pmod 3 \end{cases}$. *[3]*

2. Determine the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{\tau(n)}{n} x^n$, where $\tau : \mathbb{N}^+ \to \{-1, 1\}$ is some function that assigns one of ± 1 to each $n > 0$. *[3]*

3. Find an example of a function $\tau : \mathbb{N}^+ \to \{-1, 1\}$ which makes the interval of convergence of $\sum_{n=1}^{\infty} \frac{\tau(n)}{n} x^n$ be, respectively,

a. $(-R, R)$ **b.** $[-R, R)$ **c.** $(-R, R]$ **d.** $[-R, R]$

(where R is the radius of convergence from **2**) or show that there is no such τ. *[4]*

Bonus. Determine, as best you can, whether the following relative of the harmonic series converges or diverges: $\sum_{n=1}^{\infty} \frac{\rho(n)}{n}$, where $\rho : \mathbb{N}^+ \to \{-1, 1\}$ randomly chooses one of ± 1 for each $n > 0$, in such a way that $\lim_{n \to \infty} \frac{\rho(1) + \rho(2) + \cdots + \rho(n)}{n} = 0$. (That is, the series is "alternating on average.") *[1]*