Mathematics 3790H – Analysis I: Introduction to analysis
TRENT UNIVERSITY, Fall 2009

Quizzes

Quiz #1. Thursday, 24 September, 2009 *(10 minutes)*

The series $\sum_{n=0}^{\infty} \frac{1}{2^n} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$ sums to 2. Denote the kth partial sum of this series by $S_k = \sum_{n=0}^{k} \frac{1}{2^n} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^k}$.

1. Show that $S_k < 2$ for every $k \geq 0$. *[2]*

2, How large does k need to be to ensure that the partial sum $S_k = \sum_{n=0}^{k} \frac{1}{2^n}$ of this series is within 0.001 of 2? *[3]*

Hints: First, what, exactly, is $2 - S_k$? Second, note that $2^{10} = 1024$.

Quiz #2. Thursday, 1 October, 2009 *(10 minutes)*

You may assume that $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$ converges to $\frac{1}{1-x}$ for $|x| < 1$. Find the sum of each of the following series for $|x| < 1$:

1. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$ *[2]*
2. $\sum_{n=0}^{\infty} (n+1)x^n = 1 + 2x + 3x^2 + 4x^3 + \cdots$ *[3]*

Hints: Substitution. Calculus.

Quiz #3. Thursday, 8 October, 2009 *(10 minutes)*

1. Show that the sequence $y_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln(n)$ is decreasing. *[5]*

Quiz #4. ~~Thursday, 15~~ Monday, 19 October, 2009 *(10 minutes)*

Do *one* of questions 1 and 2.

1. Use Lagrange's Remainder Theorem to determine the number of terms of the of the partial sum for the power series expansion of $f(x) = \ln(1 + x)$ that are needed to guarantee that the partial sum is within 0.1 of $\ln(2) = \ln(1 + 1)$. *[5]*

   Hint: You may assume that the power series expansion of $f(x)$ is $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + \frac{(-1)^n x^n}{n} + \cdots$ and that $f^{(n)}(x) = \frac{(-1)^{n+1}(n-1)!}{(1+x)^n}$ for $n \geq 1$.
2. Use the Intermediate Value Theorem to show that every real number $\alpha > 0$ has a square root. *[5]*

   Hint: α has a square root if $f(x) = x^2$ takes on the value α ...

Quiz #5. Thursday, 22 October, 2009 *(10 minutes)*

1. Suppose $f(x)$ is a function that is defined for all x near 0 and is continuous at 0, and suppose c is a real number. Use the $\varepsilon - \delta$ definition of continuity to show that $g(x) = cf(x)$ is also continuous at 0. *[5]*

Quiz #6. Thursday, 12 November, 2009 *(10 minutes)*

1. Use the $\varepsilon - \delta$ definition of continuity to show that $g(x) = \frac{1}{3x-1}$ is continuous at 1. *[5]*

Take-home Quiz #7. Due on Monday, 16 November, 2009

1. Suppose $f(x)$ and g(x) are functions that are defined and continuous for all x near a, and such that $g(a) \neq 0$. Use the $\varepsilon - \delta$ definition of continuity to show that $h(x) = \frac{f(x)}{g(x)}$ is also continuous at a. *[5]*

Quiz #8. Thursday, 19 November, 2009 *(15 minutes)*

You may assume that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges and that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. Use the Comparison Test to determine whether or not each of the following series converges.

1. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ *[1.5]* 2. $\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^2}$ *[1.5]* 3. $\sum_{n=0}^{\infty} \frac{n}{n^3+1}$ *[2]*

Quiz #9. Thursday, 26 November, 2009 *(12 minutes)*

1. Use the (limit) ratio test to verify that $\displaystyle\sum_{n=0}^{\infty} \frac{\pi^n}{n!}$ converges absolutely. *[2]*
2. Use the convergence test(s) of your choice to determine whether $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$ converges absolutely, converges conditionally, or diverges. *[3]*

Quiz #10. Thursday, 3 December, 2009 *(10 minutes)*

1. Find the interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{2^n x^n}{n+1}$. *[5]*

Quiz #11. Thursday, 11 December, 2009 *(10 minutes)*

1. Show that the functions $f_n(x) = 1 + x^n$ converge uniformly to $f(x) = 1$ on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$. *[5]*