Mathematics 3790H – Analysis I: Introduction to analysis
TRENT UNIVERSITY, Fall 2009

Assignment #2 — Series Business
Due on Thursday, 8 October, 2009.

For questions **1** and **2**, assume that we know that

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

for all $x \in \mathbb{R}$.

1. Work out the power series for a^x, where a is a positive real number. *[3]*

2. Show that $e^{s+t} = e^s e^t$ by doing algebra with the appropriate power series. *[4]*

3. The modern (and Archimedean!) meaning of "the series $\sum_{i=0}^{\infty} a_i$ converges to A" is usually captured by a definition like:

($*$) $\sum_{i=0}^{\infty} a_i$ converges to A if for every $\varepsilon > 0$ there is a K such that for all $k \geq K$ we have $\left| \left(\sum_{i=0}^{k} a_i \right) - A \right| < \varepsilon$.

Archimedes himself would probably have said something more along the following lines:

($\bullet$) $\sum_{i=0}^{\infty} a_i$ converges to A if both

(1) for every $L < A$ there is a K such that for all $k \geq K$ we have $L < \left(\sum_{i=0}^{k} a_i \right)$, and

(2) for every $U > A$ there is a K' such that for all $k \geq K'$ we have $\left(\sum_{i=0}^{k} a_i \right) < U$.

Explain, in detail, why these two definitions are actually equivalent. *[3]*