Mathematics 3790H – Analysis I: Introduction to analysis
TRENT UNIVERSITY, Fall 2008

Quizzes

Quiz #1. Wednesday, 17 September, 2008. *[10 minutes]*

1. Find the sum of the series $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n = 1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \cdots$. *[2]*
2. Verify that the geometric series $\sum_{n=0}^{\infty} \frac{x^n}{2^{n+1}}$ and $\sum_{n=0}^{\infty} (x-1)^n$ are equal for any x for which they both converge. *[3]*

Quiz #2. Wednesday, 24 September, 2008. *[10 minutes]*

1. Use Newton's binomial series formula to find an infinite series which sums to $\dfrac{1}{\sqrt{2}}$. *[5]*

Quiz #3. Wednesday, 1 October, 2008. *[10 minutes]*

You may assume that $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots$ is the Taylor series of $f(x) = \frac{1}{1-x}$ at 0, and let $R_{n,0}(x) = D_n(0,x) = f(x) - 1 - x - x^2 - \cdots - x^n$ denote the nth remainder at 0.

1. Find $f^{(4)}(0)$. *[2]*
2. What does the Langrange Remainder Theorem tell you about $R_{4,0}(x)$? *[3]*

Quiz #4. Wednesday, 8 October, 2008. *[10 minutes]*

1. Use the $\varepsilon - \delta$ definition of limits to verify that $\lim_{x \to 0} x\cos(x) = 0$. *[5]*

Quiz #5. Wednesday, 29 October, 2008. *(Open book!) [10 minutes]*

1. Give an example to show that the following converse to the Mean Value Theorem is *not* true. *[5]*

   Suppose a function $f(x)$ is defined and differentiable for all x. Then, for every $x = c$, there are a and b with $a < c < b$ such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

   Explain (informally!) why your example does the job. *[5]*

Quiz #6. Wednesday, 29 October, 2008. *(Open book!) [10 minutes]*

Give an example of each of the following:

1. A function which is defined for all x and is strictly decreasing but does not satisfy the intermediate value theorem. *[1]*
2. A function which defined for all x and satisfies the intermediate value property, but is not differentiable for all x. *[2]*
3. A function which is defined for all x in $[-1, 1]$ and is continuous except at $x = 0$, satisfies the intermediate value property on $[-1, 1]$, but is unbounded on $[-1, 1]$. *[2]*

Explain (informally!) why your examples do the job.

Quiz #7. Wednesday, 5 November, 2008. *[10 minutes]*

Let $f(x) = 1 + x^2 + x^4$.

1. What is the Taylor series of $f(x)$ at $a = -1$? *[2.5]*
2. What is the Cauchy form of the remainder $R_{3,-1}(x) = f(x) - T_{3,-1}(x)$? *[2.5]*
   Note: Recall that $T_{n,a}(x)$ is the polynomial in $x - a$ consisting of the terms of degree $\leq n$ of the Taylor series of $f(x)$ at a.

Bonus. Find an explicit value for the $c \in (-1, 0)$ that appears in the Cauchy form of the remainder $R_{3,-1}(0)$. *[1]*

Quiz #8. Wednesday, 12 November, 2008. *[15 minutes]*

1. Show that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3+(-1)^n}} = \frac{1}{1^2} + \frac{1}{2^4} + \frac{1}{3^2} + \frac{1}{4^4} + \frac{1}{5^2} + \frac{1}{6^4} + \cdots$ converges. *[2]*
2. Suppose that $\sum_{n=0}^{\infty} a_n$ converges absolutely, $B > 0$, and that $|b_n| \leq B$ for each $n \geq 0$. Show that $\sum_{n=0}^{\infty} a_n b_n$ converges absolutely. *[3]*

Quiz #9. Wednesday, 19 November, 2008. *[5 minutes]*

1. Use the Ratio Test to show that $\displaystyle\sum_{n=0}^{\infty} \frac{c^n}{n!}$ converges for any $c \in \mathbb{R}$. *[5]*

Quiz #10. Wednesday, 26 November, 2008. *(Open book!)* *[7 minutes]*

1. Determine whether the series $\displaystyle\sum_{n=0}^{\infty} \frac{(2k)!}{4^k \cdot k! \cdot k!}$ converges absolutely, converges conditionally, or diverges. *[5]*

Quiz #11. Wednesday, 3 December, 2008. *[10 minutes]*

1. Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{(n+1)^3}$. *[5]*