

Mathematics 3790H – Analysis I: Introduction to analysis

TRENT UNIVERSITY, Fall 2008

Assignment #6

Due: Friday, 5 December, 2008

Suppose α , β , and γ are any real numbers not in $\mathbb{Z}^{\leq 0} = \{0, -1, -2, \dots\}$, and consider the following power series:

$$\begin{aligned} & 1 + \frac{\alpha \cdot \beta}{1 \cdot \gamma} x + \frac{\alpha(\alpha + 1) \cdot \beta(\beta + 1)}{1 \cdot 2 \cdot \gamma(\gamma + 1)} x^2 + \frac{\alpha(\alpha + 1)(\alpha + 2) \cdot \beta(\beta + 1)(\beta + 2)}{1 \cdot 2 \cdot 3 \cdot \gamma(\gamma + 1)(\gamma + 2)} x^3 + \dots \\ & = 1 + \sum_{n=1}^{\infty} \frac{\alpha(\alpha + 1) \dots (\alpha + n - 1) \cdot \beta(\beta + 1) \dots (\beta + n - 1)}{n! \cdot \gamma(\gamma + 1) \dots (\gamma + n - 1)} x^n \end{aligned}$$

This is what used to be called a hypergeometric series before the more general definition used in our textbook came along.

1. Why are the constants α , β , and γ not allowed to be 0 or any negative integer in the definition above? [1]
2. Determine for which values of x this series respectively converges absolutely, converges conditionally, and diverges. [9]