Mathematics 3790H – Analysis I: Introduction to analysis
TRENT UNIVERSITY, Fall 2008

Assignment #5
Due: Friday, 21 November, 2008

Recall that the harmonic series, $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$, diverges, while its close relation, the alternating harmonic series, $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$, converges.

Determine, as best you can, whether each of the following relatives of the harmonic series converges or diverges.

1. $1 + \frac{1}{2} - \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \frac{1}{6} - \frac{1}{7} - \frac{1}{8} + \frac{1}{9} + \cdots = \sum_{n=1}^{\infty} \frac{\alpha(n)}{n}$, where $\alpha : \mathbb{N}^+ \to \{-1, 1\}$ is given by $\alpha(n) = \begin{cases} +1 & n = 1 \text{ or } 2 \pmod 4 \\ -1 & n = 3 \text{ or } 4 \pmod 4 \end{cases}$. *[3]*

2. $1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} - \frac{1}{9} + \cdots = \sum_{n=1}^{\infty} \frac{\beta(n)}{n}$, where $\beta : \mathbb{N}^+ \to \{-1, 1\}$ is given by $\beta(n) = \begin{cases} +1 & n = 1 \pmod 3 \\ -1 & n \neq 1 \pmod 3 \end{cases}$. *[3]*

3. $1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \frac{1}{5} - \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} - \frac{1}{9} - \frac{1}{10} - \frac{1}{11} + \frac{1}{12} + \cdots = \sum_{n=1}^{\infty} \frac{\gamma(n)}{n}$, where $\gamma : \mathbb{N}^+ \to \{-1, 1\}$ delivers a block $+1$s of length k followed by a block of -1s of length k for $k = 1, 2, 3, \ldots$. *[3]*

4. $\sum_{n=1}^{\infty} \frac{\tau(n)}{n}$, where $\tau : \mathbb{N}^+ \to \{-1, 1\}$ randomly chooses, with equal probability, one of $+1$ and -1 for each $n > 0$. *[1]*