Mathematics 3790H – Analysis I: Introduction to analysis
Trent University, Fall 2008

Assignment #4
Math Trek: Dilithium? No, dilogarithm!
Due: Friday, 7 November, 2008

The *dilogarithm* function, $\mathrm{Li}_2(x)$, is usually defined as the sum of an infinite series:

$$\mathrm{Li}_2(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2} = x + \frac{x^2}{4} + \frac{x^3}{9} + \frac{x^4}{16} + \dots$$

To answer the questions below you will probably want to review the basic information on convergence of series from your first-year calculus text, especially the (simplest forms of the) Comparison Test and the Integral Test.

1. Show that the series defining $\mathrm{Li}_2(x)$ converges for all x with $-1 \leq x \leq 1$. *[3]*

2. How is the dilogarithm function related to the natural logarithm function? *[3]*

3. Denote the kth remainder term at 0 of the dilogarithm function by:

$$R_{k,0}(x) = \mathrm{Li}_2(x) - \sum_{n=1}^{k} \frac{x^n}{n^2} = \mathrm{Li}_2(x) - \left(x + \frac{x^2}{4} + \frac{x^3}{9} + \cdots + \frac{x^k}{k^2} \right)$$

Show that for any $\varepsilon > 0$ there is an $K > 0$ such that for any $k \geq K$, $|R_{k,0}(x)| < \varepsilon$ for all x with $-1 \leq x \leq 1$. *[4]*