Mathematics 3790H – Analysis I: Introduction to analysis
TRENT UNIVERSITY, Fall 2008

Assignment #2
The integral form of the remainder of a Taylor series
Due: Wednesday, 8 October, 2008

In what follows, let us suppose that a is a real number and $f(x)$ is a function such that $f^{(n)}(x)$ is defined and continuous for all $n \geq 0$ and all values of x we may encounter. Recall that for $n \geq 0$, the Taylor polynomial of degree n of $f(x)$ at a is

$$\begin{aligned} T_{n,a}(x) &= \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k \\ &= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n \,, \end{aligned}$$

and that the corresponding remainder term is

$$R_{n,a}(x) = f(x) - T_{n,a}(x)\,.$$

1. Use the Fundamental Theorem of Calculus to show that

$$R_{0,a}(x) = \int_a^x f'(t)\,dt\,. \quad \textit{[1]}$$

2. Use the formula in **1** and integration by parts to show that

$$R_{1,a}(x) = \int_a^x f''(t)(x-t)\,dt\,. \quad \textit{[2]}$$

Hint: Use the parts $u = f'(t)$ and $v = t - x$ ...

3. Use the formula in **2** and integration by parts to show that

$$R_{2,a}(x) = \int_a^x \frac{f^{(3)}(t)}{2}(x-t)^2\,dt\,. \quad \textit{[2]}$$

4. Find an integral formula for $R_{n,a}$ and use induction to show that it works. *[5]*