Math 356H Assignment #5 Solutions

1. Chapter 10, #24.

Source	Df	SS	MS	F
Groups	3-1=2	152.18	76.09	5.56
Error	74-3=71	970.96	13.68	
Total	74-1=73	1123.14		

Since $5.56 \geq 4.94 \approx F_{.01,2,71}$, we reject H_0 at $\alpha = .01$. Hence not all means are the same.

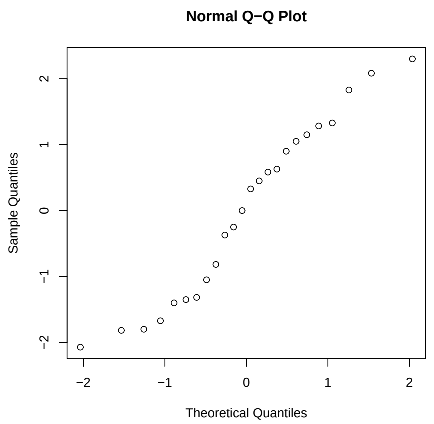
2. Chapter 10, #27.

ANOVA

Source	SS	df	MS	F	Pr(¿F)
Brand	23.49571429	3	7.831904762	3.749330355	0.02755
Error	41.77761905	20	2.088880952		
Total	65.27333333	23			

(a) Since $P = .02755 < .05$, we reject H_0. (Or from tables: since $F_{.01,3,20} = 4.94 > 3.75 > F_{.05,3,20} = 3.1$, $.01 < P < .05$ and we reject H_0 at $\alpha = .05$.) Hence not all means can be considered equal.

(b) The normal probability plot of residuals appears here. It indicates that the distribution of residuals could be normal, so the normality assumption is plausible.



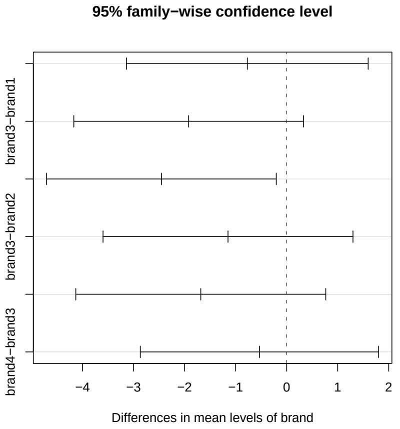
(c) t Tukey multiple comparisons of means
95% family-wise confidence level

brands	diff	lwr	upr	p adj
brand2-brand1	-0.7714286	-3.140109	1.5972517	0.7989522
brand3-brand1	-1.9214286	-4.172021	0.3291639	0.1115525
brand4-brand1	-2.4547619	-4.705354	-0.2041694	0.0294477
brand3-brand2	-1.1500000	-3.599546	1.2995457	0.5650524
brand4-brand2	-1.6833333	-4.132879	0.7662124	0.2501669
brand4-brand3	-0.5333333	-2.868884	1.8022169	0.9180643

We see that there are significant differences only between brand 1 and brand 4. These differences can be seen in the plot below:

3. Rat poison is normally made by mixing its active chemical ingredients with ordinary cornmeal. In many urban areas, though, rats can find food that they prefer to cornmeal, so the poison is left untouched. One solution is to make the cornmeal more palatable by adding food supplements such as peanut butter or meat. Doing that is effective, but the cost is high and the supplements spoil quickly.

In Milwaukee, a study was carried out to see whether artificial food supplements might be a workable compromise. For five two-week periods, thirty-two hundred baits were placed



around garbage-storage areas: eight hundred consisted of plain cornmeal; a second eight hundred had cornmeal mixed with artificial butter-vanilla flavoring; a third eight hundred contained corneal mixed with artificial roast beef flavoring; and the remaining eight hundred were cornmeal mixed with artificial bread flavoring.

The following table lists, for each survey, the percentage of each type of bait that was eaten.

Survey number	Plain	Butter vanilla	Roast beef	Bread
1	13.8	11.7	14.0	12.6
2	12.9	16.7	15.5	13.8
3	25.9	29.8	27.8	25.0
4	18.0	23.1	23.0	16.9
5	15.2	20.2	19.0	13.7

(a) What is the factor? How many levels are in this design?

The factor is the type of bait. There are 4 levels of the factor of interest.

(b) What is the blocking factor?

Survey number is the blocking factor.

(c) Do the rats show any preferences for the different flavors?

The ANOVA table is given below (rows are the blocking factor and columns are the factor):

ANOVA

Source	SS	df	MS	F	P-value	F crit
Rows	495.322	4	123.8305	49.92829783	2.20835E-07	3.259166727
Columns	56.378	3	18.79266667	7.577178953	0.004183558	3.490294821
Error	29.762	12	2.480166667			
Total	581.462	19				

With a P-value of .0041, we reject the null hypothesis of no preference, so the rats do show a preference for different flavours.

(d) Were the blocks helpful in reducing the error sum of squares?

Yes, there is significance of the blocking factor.

(e) If a follow-up study were to be done, comparing these same baits, should a completely randomized design or a randomized block design be used?

A randomized block design should be used because of the significance of using blocks.

4. A particular county employs three assessors who are responsible for determining the value of residential property in the county. To see whether these assessors differ systematically in their assessments, 5 houses are selected, and each assessor is asked to determine the market value of each house. With factor A denoting assesors ($I = 3$) and blocking factor B denoting houses ($J = 5$), suppose $SSA = 11.7$, $SSB = 113.5$, and $SSE = 25.6$.

(a) Test the hypothesis that there are no systematic differences among assessors.

Assessors correspond to the A factor, so we test the hypothesis of no differences using the F statistic $MSA/MSE = (11.7/2)/(25.67/8) = 1.82$, which has a P-value of .2231, so that the null hypothesis is not rejected at $\alpha = .05$. Hence there is no systematic difference among assessors. (Alternatively, we compare with the critical $F_{.05,2,8} = 4.459$, so we fail to reject.)

(b) Explain why a randomized block experiment with only 5 houses was used rather than a one-way ANOVA experiment involving a total of 15 different houses with each assessor asked to assess 5 different houses (a different group of 5 for each assessor).

Blocking was introduced to reduce variance. Variability due to house effect could have made us believe that there are significant differences in assessors.

(c) Suppose now that the houses had actually been selected at random from among those of a certain age and size, so that factor B is random rather than fixed. Test $H_0 : \sigma_B^2 = 0$ using a level .01 test.

The test proceeds in exactly the same way as with the fixed factor case, except that the null hypothesis is that the variance is 0 rather than the block effect is zero. We test using $F = MSB/MSE = (113.5/4)(25.67/8) = 8.843$, which has a P-value of .0094, so that we reject the hypothesis of no effect due to blocking. Hence blocking was a good idea! (Alternatively, we compare to the critical $F_{.05,4,8} = 3.838$, so that again we reject H_0.)

5. Chapter 11, #16.

(a) ANOVA

Source	Df	SS	MS	F
A	2	30,763.0	15,381.50	3.79
B	3	34,185.6	11,395.20	2.81
AB	6	43,581.2	7263.53	1.79
Error	24	97,436.8	4059.87	
Total	35	205,966.6		

(b) $F_{AB} = 1.79 < 2.51 = F_{.05,6,24}$, so H_{0AB} cannot be rejected, and we conclude that no interaction is present.

(c) $F_A = 3.79 > 3.40 = F_{.05,2,24}$, so we reject H_{0A} at level $\alpha =$, and there is an effect from factor A.

(d) $F_B = 2.81 < 3.01 = F_{.05,3,24}$, so H_{0B} is not rejected, and there is not effect from factor B.

(e) $Q_{.05,3,24} = 3.53$, $w = 3.53\sqrt{\frac{4059.87}{12}} = 64.93$. Thus mean 3 (3960.02) and mean 2 (4010.88) can be considered equal, and mean 2 and mean 1 (4029.10) can be considered equal. Only times 2 and 3 yield significantly different strengths.

6. Chapter 11, #20.

ANOVA

Source	Df	SS	MS	F	$F_{.01}$
A	1	13,338.89	13,338.89	192.09	9.93
B	2	1244.44	622.22	8.96	6.93
AB	2	44.45	22.23	.32	6.93
Error	12	833.33	69.44		
Total	17	15,461.11			

Clearly, $F_{AB} = .32$ is not significant, so H_{0AB} is not rejected. Both H_{0A} and H_{0B} are rejected, since they are both greater than the respective critical values. Both phosphor type and glass type significantly affect the current necessary to produce the desired level of brightness.