Mathematics 1350H – Linear algebra I: matrix algebra

TRENT UNIVERSITY, Fall 2009

MATH 1350H Test

4 November, 2009

Time: 50 minutes

Instructions

- *Show all your work.* Legibly, please!
- *If you have a question, ask it!*
- Use the back sides of the test sheets for rough work or extra space.
- You may use a calculator and an aid sheet (or an annotated *Formula for Success*).

1. Consider the line passing through the points $(1, 0, 0)$ and $(2, 1, 0)$.

a. Sketch this line. *[2]*

b. Find a parametric description of this line. *[4]*

c. What is the acute angle between this line and the plane given by $y + z = 1$? *[4]*

2. Consider the following system of linear equations:

$$\begin{array}{rcrcrcr} x & + & y & + & z & = & 6 \\ 2x & - & y & + & z & = & 3 \\ 3x & + & y & - & z & = & 2 \end{array}$$

a. Find the solution(s), if any, of this system of equations. *[7]*

b. What does your answer to **a** tell you about some planes? *[1.5]*

c. What does your answer to **a** tell you about some vectors? *[1.5]*

3. Do any *two (2)* of **a**–**c**. *[10 = 2 × 5 each]*

a. Find a linear equation for the plane given by the vector-parametric equation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

b. Sketch the plane $x + 2y + 3z = 6$.

c. Find the shortest distance from the point $(1, 1, 2)$ to the plane $x + y + z = 1$.

4. Do any *two (2)* of **a**–**c**. *[10 = 2 × 5 each]*

a. Why isn't every vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ in $\text{Span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \right\}$?

b. Compute $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^8 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

c. Find a 2×3 matrix $\mathbf{A}$ such that $\mathbf{A}\mathbf{A}^T = \mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

[Total = 40]