

Mathematics 1350H – Linear algebra I: matrix algebra

TRENT UNIVERSITY, Fall 2009

Assignment #3

Due on Friday, 23 October, 2009.

Linear optimization

In this assignment we will deal with the solid whose faces are (parts of) the planes given by the equations $x = 0$, $y = 0$, $z = 0$, $-2x + y + z = 3$, $x - 2y + z = 3$, $x + y - 2z = 3$, $-3x + 2y + 2z = 7$, $2x - 3y + 2z = 7$, and $2x + 2y - 3z = 7$. Another way to look at this solid is as the set of points with coordinates (x, y, z) which satisfy *all* of the following nine inequalities: $x \geq 0$, $y \geq 0$, $z \geq 0$, $-2x + y + z \leq 3$, $x - 2y + z \leq 3$, $x + y - 2z \leq 3$, $-3x + 2y + 2z \leq 7$, $2x - 3y + 2z \leq 7$, and $2x + 2y - 3z \leq 7$.

1. Find the coordinates of all of the vertices of this solid and make as accurate a sketch as you can of it. [6]
2. Find the maximum value of the function $f(x, y, z) = 2x + 10y - 9z$ on this solid and determine at which point(s) of the solid this maximum occurs. [4]

Note: In this context the inequalities defining the solid are called *linear constraints*. Problems involving the optimization of a linear function subject to linear constraints arise often enough to be pretty important in the real world. If you're interested in the methods used to solve such problems, try taking MATH-COIS 3350H – *Linear optimization*. (Still titled as *Linear programming* in the *University Calendar* for some reason.) MATH 1350H is the only prerequisite.

JENNET . . . Poor father. In the end he walked
In Science like the densest night. And yet
He was greatly gifted.
When he was born he gave an algebraic
Cry; at one glance measured the cubic content
Of that ivory cone his mother's breast
And multiplied his appetite by five.
So he matured by a progression, gained
Experience by correlation, expanded
Into a marriage by contraction, and by
Certain physical dynamics
Formulated me. And on he went
Still deeper into the calculating twilight
Under the twinkling of five-pointed figures
Till Truth became for him the sum of sums
And Death the long division. My poor father.
What years and powers he wasted.
He thought he could change the matter of the world
From the poles to the simultaneous equator
By strange experiment and by describing
Numerical parabolas.

From *The Lady's Not For Burning*, by Christopher Fry.
(My favourite play! *Stefan*)