Mathematics 1350H – Linear algebra I: matrix algebra
TRENT UNIVERSITY, Fall 2008

MATH 1350H Test

3 November, 2008
Time: 50 minutes

Instructions

- *Show all your work.* Legibly!
- *If you have a question, ask it!*
- Use the back sides of the test sheets for rough work or extra space.
- You may use a calculator, and either (both sides of) one 8.5×11 aid sheet or a copy (annotated as you like) of *Formula for Success*.

1. Consider the points $(2,0,0)$, $(0,2,0)$, and $(0,0,2)$ in $\mathbb{R}^3$.

a. Find a parametric description of the line passing through the first two points. *[3]*

b. Find a linear equation describing the plane passing through all three points. *[4]*

c. Sketch the part of the plane in **b** that lies in the first octant. *[3]*

2. Use the Gauss-Jordan method to find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 1 & -1 & 10 \end{bmatrix}$, if one exists. *[10]*

3. Do any *two* of parts **a**, **b**, **c**. *[10 = 2 × 5 each]*

a. Suppose $\mathbf{B}$ is an $n \times n$ matrix which is invertible and for which $\mathbf{B}^2 = \mathbf{B}$. Show that $\mathbf{B} = \mathbf{I}_n$, the $n \times n$ identity matrix.

b. Find the (shortest) distance from the point $P = (1,0,0)$ to the line ℓ given by the parametric equations $x = 1$, $y = 1 - 2t$, and $z = 1 + 3t$.

c. Can there be four planes in $\mathbb{R}^3$ which are each perpendicular to the other three? If so, give an example; if not, explain why not.

4. Let $\mathbf{a} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$, and $\mathbf{d} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$. Do *one* of parts $\triangle$ or $\square$. *[10]*

$\triangle$. Determine whether $\mathbf{d}$ is in $\text{Span}\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ or not.

$\square$. Determine whether **a**, **b**, **c**, and **d** are linearly independent or not.

[Total = 40]