Mathematics 1350H – Linear algebra I: matrix algebra

TRENT UNIVERSITY, Fall 2008

Assignment #2

Due on Friday, 3 October, 2008.

Shifty business

Suppose $\mathbf{a} = [a_1, a_2, \ldots, a_n]$ is an n-place row vector. The *left shift* of $\mathbf{a}$ by k places (where $0 \leq k < n$) is the vector $\sigma_k(\mathbf{a}) = [a_{k+1}, a_{k+2}, \ldots, a_n, a_1, a_2, \ldots, a_k]$. For example, here is a left shift by 2 places of a 6-place vector:

$$\sigma_2\left([3, -1, 0, 5, 1, -2]\right) = [0, 5, 1, -2, 3, -1]$$

Note that for any vector $\mathbf{a}$, $\sigma_0\left(\mathbf{a}\right) = \mathbf{a}$.

1. Explain why $\sigma_k(s\mathbf{a}) = s\sigma_k(\mathbf{a})$ for any n-place vector $\mathbf{a}$, integer k with $0 \leq k < n$, and scalar s. *[1.5]*
2. Explain why $\sigma_k(\mathbf{a} + \mathbf{b}) = \sigma_k(\mathbf{a}) + \sigma_k(\mathbf{b})$ for any n-place vectors $\mathbf{a}$ and $\mathbf{b}$, and any integer k with $0 \leq k < n$. *[1.5]*
3. Is it true that $\sigma_k\left(\sigma_\ell(\mathbf{a})\right) = \sigma_{k+\ell}(\mathbf{a})$? Explain why or why not. *[2]*
4. How does the left shift operator interact with the dot product? *[2]*
5. Find a vector $\mathbf{a}$ with as many places as you can such that each entry of $\mathbf{a}$ is either $+1$ or -1, and such that for every left shift by $k > 0$ places we have $|\mathbf{a} \cdot \sigma_k(\mathbf{a})| \leq 1$. *[3]*

Equation Limericks

$$\frac{12 + 144 + 20 + 3\sqrt{4}}{7} + 5 \cdot 11 = 9^2 + 0$$

A dozen, a gross, and a score,
Plus three times the square root of four,
Divided by seven,
Plus five times eleven,
Is nine squared and not a bit more!

Posted to `sci.math` by Rajeev Krishnamoorthy on 1992.04.23.

$$\int_1^{\sqrt{3}} t^2\,dt \cdot \cos\left(\frac{3\pi}{9}\right) = \log\left(\sqrt[3]{e}\right)$$

The integral t squared dt
from one to the square root of three,
times the cosine
of three π over nine
is the log of the cube root of e.

Posted to `sci.math` by Gerald A. Edgar on 1992.04.18. (Slightly modified since.)

Both equations are true, by the way. Can you devise an original (true!) equation limerick? If you do, save it up for later on!