Mathematics 135H – Linear algebra I: matrix algebra
TRENT UNIVERSITY, Fall 2007

MATH 135H Test

2 November, 2007
Time: 50 minutes

Instructions

- *Show all your work.* Legibly!
- *If you have a question, ask it!*
- Use the back sides of the test sheets for rough work or extra space.
- You may use a calculator, and either (both sides of) one 8.5×11 aid sheet or a copy (annotated as you like) of *Formula for Success*.

1. Consider the planes defined by the equations $x + 2y + z = 6$ and $2x + y + z = 6$ in three-dimensional space.

a. Sketch the parts of the two planes and the line in which they intersect that lie in the first octant (*i.e.* where $x \geq 0$, $y \geq 0$, and $z \geq 0$). *[4]*

b. Give a parametric description of the line in which the two planes intersect. *[3]*

c. Determine whether the two planes are parallel, perpendicular, or neither. *[3]*

2. Let $\mathbf{a} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 8 \\ -2 \\ 2 \end{bmatrix}$, and $\mathbf{c} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$.

a. Determine whether $\mathbf{d} = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$ is in $\text{Span}\{\, \mathbf{a}, \mathbf{b}, \mathbf{c} \,\}$ or not. *[5]*

b. Determine whether $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are linearly dependent or independent. *[5]*

3. Consider the following system of linear equations.

$$\begin{array}{ccccccc} x & & & + & z & = & 1 \\ x & & & - & z & = & 0 \\ x & + & cy & + & z & = & 0 \end{array}$$

a. Find the solution(s), if any, of the given system in terms of c, the coefficient of y in the third equation. *[6]*

b. For which values of c are there: *i.* No solutions? *ii.* Exactly one solution? *iii.* Many solutions? Explain why in each case. *[4]*

4. Let $\mathbf{B} = \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ 1 & 3 \end{bmatrix}$ and $\mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. In each of **a**–**c**, find an example of a 2×3 matrix $\mathbf{A}$ satisfying the given matrix equation or explain why there is no such $\mathbf{A}$.

a. $\mathbf{AB} = \mathbf{I}_2$. *[4]* **b.** $\mathbf{BA} = \mathbf{I}_2$. *[3]* **c.** $\mathbf{AA}^T = \mathbf{I}_2$. *[3]*

[Total = 40]