

## Mathematics 135H – Linear algebra I: matrix algebra

TRENT UNIVERSITY, Fall 2007

### Solutions to Assignment #1

Answering the bonus problem first, EXCELSIOR is the first “Knot” of Lewis Carroll’s book *A Tangled Tale*, which was originally a magazine serial. Lewis Carroll was the pen name of Charles Lutwidge Dodgson, who was a lecturer in mathematics at Oxford, among other things. He is best remembered nowadays for having written *Alice in Wonderland* and *Through the Looking Glass*. Here are the solutions Carroll gives in *A Tangled Tale*:

*Problem.*—“Two travelers spend from 3 o’clock till 9 in walking along a level road, up a hill, and home again: their pace on the level being 4 miles an hour, up hill 3, and down hill 6. Find distance walked: also (within half an hour) time of reaching top of hill.”

*Answer.*—“24 miles: half past 6.”

*Solution.*—A level mile takes  $\frac{1}{4}$  of an hour, up hill  $\frac{1}{3}$ , down hill  $\frac{1}{6}$ . Hence to go and return over the same mile, whether on the level or on the hill-side, takes  $\frac{1}{2}$  an hour. Hence in 6 hours they went 12 miles out and 12 back. If the 12 miles had been nearly all level, they would have taken a little over 3 hours; if nearly all up hill, a little under 4. Hence  $3\frac{1}{2}$  hours must be within  $\frac{1}{2}$  an hour of the time taken in reaching the peak; thus, as they started at 3, they got there within  $\frac{1}{2}$  an hour of  $\frac{1}{2}$  past 6.

Notice that Carroll does not make the assumption that the travelers return by the same route explicit, though he clearly uses it in his solution below. He also assumes – unnecessarily! – that the path the travelers took consists of a stretch of level road followed by a hill, so his solution is not quite as comprehensive as it might be. In any event, our problem **2** is not answered by Carroll’s solution. Here is a sketch of one way to answer it for distance:

Suppose we let  $u$ ,  $d$ , and  $\ell$  denote the time (in hours) spent going uphill, downhill, and on level ground over the round trip. Then the total distance covered is  $D = 3u + 6d + 4\ell$  miles. Since the travelers return after six hours, we know that  $6 = u + d + \ell$ . As they also retrace their path, we also know that the overall uphill distance is the same as the overall downhill distance, so  $3u = 6d$  (or  $u = 2d$ ). So we have three equations in four unknowns we want to solve for  $D$ .

A solution to the distance problem can now be obtained as follows:

Using  $u = 2d$ , rewrite the previous two equations in terms of  $d$  and  $\ell$  alone, giving  $6 = 3d + \ell$  and  $D = 12d + 4\ell$ . There are still more unknowns than equations, but this particular problem can now be solved with a cheap trick:

$$D = 12d + 4\ell = 4(3d + \ell) = 4 \cdot 6 = 24 \text{ miles}$$

The problem of determining the range of possibilities for the time the travelers were at the peak is left to the reader ...

#### REFERENCES

1. *Pillow Problems* and *A Tangled Tale*, by Lewis Carroll (*Charles Lutwidge Dodgson*). Macmillan and Co., 1895 and 1885, respectively. Reprinted together by Dover Publications, New York. (ISBN 0-486-20493-6) A link to the full text of *A Tangled Tale* can be found at:

<http://www.trentu.ca/mathematics/sb/carroll/>