Mathematics 135H – Linear algebra I: matrix algebra
TRENT UNIVERSITY, Fall 2007

Assignment #4
Due on Friday, 23 November, 2007.

Possible limits of matrices?*

Suppose $\mathbf{A}$ is a $k \times k$ matrix for some $k \geq 2$. Consider the sequence of matrices: $\mathbf{A}$, $\mathbf{A}^2$, $\mathbf{A}^3$, ... What would it really mean to say that this sequence of matrices has some matrix $\mathbf{B}$ as its limit? That is really beyond the scope of this course, but there is one property that a limit ought to have that we can explore a little. In particular, if $\mathbf{B}$ were a limit of the sequence, it also ought to be, for any fixed $m > 0$, a limit of the sequence $\mathbf{A}\mathbf{A}^m$, $\mathbf{A}^2\mathbf{A}^m$, $\mathbf{A}^3\mathbf{A}^m$, ... (Note that this is the same sequence as $\mathbf{A}^m\mathbf{A}$, $\mathbf{A}^m\mathbf{A}^2$, $\mathbf{A}^m\mathbf{A}^3$, ...) We then ought to get that $\mathbf{B} = \mathbf{B}\mathbf{A}^m = \mathbf{A}^m\mathbf{B}$. This leads to the following definition:

A $k \times k$ matrix $\mathbf{B}$ *absorbs* the $k \times k$ matrix $\mathbf{A}$ if $\mathbf{B}\mathbf{A}^m = \mathbf{A}^m\mathbf{B} = \mathbf{B}$ for every $m > 0$.

1. Verify that $\mathbf{0}_k$ absorbs $\mathbf{A}$, for any $k \times k$ matrix $\mathbf{A}$. *[2]*

2. Find an example of a $k \times k$ matrix $\mathbf{A} \neq \mathbf{0}_k$ such that $\mathbf{A}$ absorbs itself. *[2]*

3. Find an example of a $k \times k$ matrix $\mathbf{A} \neq \mathbf{0}_k$ such that $\mathbf{0}_k$ is the *only* $k \times k$ matrix that absorbs $\mathbf{A}$. *[3]*

4. Suppose the $\mathbf{A}$ is a $k \times k$ matrix which is absorbed by a matrix $\mathbf{B}$ which has an inverse. Show that it must be the case that $\mathbf{A} = \mathbf{I}_k$. *[3]*

Note: In **2** and **3**, it suffices to find an example for a particular $k \geq 2$, while in **1** and **4** you should try to give an argument that works for any $k \geq 2$. Of course, in both problems you must verify that your example does the job.

$$\frac{12 + 144 + 20 + 3\sqrt{4}}{7} + 5 \cdot 11 = 9^2 + 0$$

A dozen, a gross, and a score,
Plus three times the square root of four,
  Divided by seven,
  Plus five times eleven,
Is nine squared and not a bit more!

Posted to `sci.math` in April 1995 by Ralph Ray Craig. This is an example of a rather specialised poetical form, the equation limerick.

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* Limits?! What are limits doing here? You'd think you'd be safe from calculus in a linear algebra course!