Mathematics 135H – Linear algebra I: matrix algebra

TRENT UNIVERSITY, Fall 2007

Assignment #3

Due on Friday, 19 October, 2007.

Linear constraints and optimization

In this assignment we will deal with the solid whose faces are (parts of) the planes given by the equations $x = 0$, $y = 0$, $z = 0$, $x + y = 10$, $x + z = 10$, $y + z = 10$, and $x + y + z = 14$. Another way to look at this solid is as the set of points with coordinates (x, y, z) (in three-dimensional Cartesian space) which satisfy *all* of the following seven inequalities: $x \geq 0$, $y \geq 0$, $z \geq 0$, $x + y \leq 10$, $x + z \leq 10$, $y + z \leq 10$, and $x + y + z \leq 14$.

1. Find the coordinates of all of the vertices of this solid and make as accurate a sketch as you can of it. *[6]*

2. Find the maximum value of the function $f(x, y, z) = 2x + 2y + 3z$ on this solid and determine at which point(s) of the solid this maximum occurs. *[4]*

Note: In this context the inequalities defining the solid are called *linear constraints*. Problems involving the optimization of a linear function subject to linear constraints arise often enough to be pretty important in the real world. If you're interested in the methods used to solve such problems, you might consider taking *Mathematics-Science 335H: Linear programming*; the only prerequisite is MATH 135H.

A Joke

Some academics relaxing in a common room are asked whether all odd numbers greater than one are prime.

The physicist proceeds to experiment — 3 is prime, 5 is prime, 7 is prime, 9 doesn't seem to be prime, but that might be an experimental error, 11 is prime, 13 is prime — and concludes that the experimental evidence tends to support the hypothesis that all odd numbers are prime.

The engineer, not to be outdone by a physicist, also proceeds by experiment – 3 is prime, 5 is prime, 7 is prime, 9 is prime, 11 is prime, 13 is prime, 15 is prime – and concludes that all odd numbers must be prime.

The statistician checks a randomly chosen sample of odd numbers — 17 is prime, 29 is prime, 41 is prime, 101 is prime, 269 is prime — and concludes that it is probably true that all odd numbers are prime.

The physicist observes that other experiments have confirmed his conclusion, but the mathematician sneers at "mere examples"; and posts the following: *3 is prime. By an easy argument which is left to the reader, it follows that all odd numbers greater than one are prime.*

The chemist observes that the periodic table gives the answer: 3 is lithium, 5 is boron, 7 is nitrogen, 9 is fluorine, 11 is sodium, ... Since elements are indivisible — nuclear fission being uncommon in chemistry labs — these are all prime. (And the same is true for even numbers too!).

The economist notes that 3 is prime, 5 is prime, 7 is prime, but 9 isn't prime, and exclaims, "Look! The prime rate is dropping!"

The computer scientist goes off to write a program to check all the odd numbers. Its output reads:

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3 is prime.
3 is prime.
3 is prime.
    ⋮
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The cultural studies professor [cultician? cultist?!] objects that one shouldn't refer to numbers as odd because they might be offended or as prime because the term implies favouritism, and the theologian concurs since all numbers must be equal before God.